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A COMPARISON OF VARIANCE-CONTROLLED AND NON-CONTROLLED PRINCIPAL COMPONENTS

THESIS

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THESIS

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Preface

The purpose of the research presented in this paper was to examine the effects of applying the technique of control variates on the principal components of the outputs of a given simulation model. In this research a combination of data and variance reduction was explored. This type of experimentation may be generalizable to any simulation model.

The percentage of total variance explained by the principal components combined with scatter plots of the principal components served as the "metrics" for comparison in this study. Although other measures of effectiveness may have been used, the metrics used herein were adequate to explain the findings for this study. The experimentation done in this thesis should be extended, as it could be of significant value to analysts comparing attributes of similar systems.

In performing the experimentation and writing this thesis, I have had a great deal of help from others. I am deeply indebted to my advisor, Maj. Kenneth W. Bauer, for the awesome patience he showed during this thesis experience. I also wish to thank Dr. James W. Chrissis, my reader, for plowing through this document in search of innumerable "faux pas." Many thanks are also owed to Capt. Alan Gigliotti for his assistance in the helping me understand the ways of computers and the intricacies of the many programs we both worked with for our respective theses. Finally, I would like to acknowledge Anthony P. Cruz, my "raison d' etre", for the extraordinary understanding and support he showed during this period.

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Thesis

Abstract

The purpose of this research was to examine the effects of applying the technique of control variates on the principal components of a given model.

The investigation was done by comparing three sets of data as follows:

1) The set of principal components of the outputs of the model on which no variance reduction has been applied,

2) The set of principal components of variance-controlled outputs--control variates was performed prior to principal components analysis being done.

3) The set of variance-controlled principal components--control variates was performed on the principal components of the outputs.

The comparison of the effects was carried out by examining the percentage of variance explained by the principal components and by reviewing the scatter plots of the first two principal components.

A COMPARISON OF VARIANCE-CONTROLLED AND NON-CONTROLLED PRINCIPAL COMPONENTS

I. Background

Experimental Design

Systems differ from each other in terms of values of system parameters and input variables. These varying parameters and variables are called factors. For example, a model using estimated research and development (R&D) costs, production costs and budgeted funds is used for life cycle costing. The parameters and variables include the costs and budgeted funds and the distributions that specify these costs and budgeted funds.

In order to find the effect of a factor on a system, the factor must be varied or analyzed at different levels. (A level is a particular value of a factor.)

Often, models contain numerous factors. A problem of analysis results when the number of these factors is high. The number of factor-level combinations may become cumbersome or even prohibitive and inhibit clear analysis. For example, 7 factors with each factor having 2 levels lead to 2^7 or 128 combinations. Limiting the number of combinations that will actually be analyzed can be done through experimental design. An experimental design determines a subset of these factor-level combinations that may be able to represent the significant effects of all the combinations.

Control Variates

Another problem that exists when analyzing models is that large variances of the outputs can limit the utility of outputs. These imprecise outputs then often are used for other higher purposes such as planning. For example, the outputs of a life cycle model are used for planning funding. Minimizing or controlling the variances of the outputs would lead to more meaningful and accurate planning.

Methods to reduce the variance of a variable are called variance reduction techniques (VRT). VRTs are techniques that replace the original sampling procedure (that generated the variable) by another procedure that yields the same expected value of the variable but with a smaller variance. One such VRT is the method of control variates.

Control variates accomplish variance reduction by taking advantage of the correlation, if any, between the input and response variables. Generally, the greater the correlation, the greater the variance reduction.

Principal Components Analysis

Large variance in the output is not the only significant problem in modeling systems. Often the outputs of a model can be incomprehensible because of the great number of output variables that need to be interpreted. Data reduction procedures may be the solution to these types of problems. Specifically, the technique of principal components analysis may be applied.

Principal components analysis takes an original set of variables and transforms it into a smaller set. The variables in the smaller set are linear combinations of the variables in the original set and explain much of the variance of the total data that was contained in the original set. The objective of principal components analysis is to produce as few as possible of the principal components, the

variables in the smaller set, that contains as much of the information of the data in the original set .

Objective

Little research has been accomplished in the area of experimental design with variance reduction and principal components analysis as applied to large models. The research that has been completed has mainly been limited to small classroom-size models.

The objective of this research was to examine the effects of performing the technique of control variates on the principal components of a given model. Before the technique of control variates and principal components analysis can be accomplished, data on which the technique and the analysis are applied must first be created. The data that were created were the outputs of a given model. The outputs that were created were required to be representative of all the outputs the model can produce. Hence, experimental design was required to specify the significant factors that produced the representative outputs.

The comparison of the effects of control variates and principal components was accomplished by comparing three sets of data. The three sets of data were as follows:

1) The set of principal components of the outputs of the model on which no variance reduction had been applied,

2) The set of principal components of variance-controlled outputs--control variates was performed before principal components analysis was accomplished.

3) The set of variance-controlled principal components--control variates was performed on the principal components of the outputs.

The comparison of the effects was done by examining the percentage of variance explained by the principal components and by reviewing the scatter plots of two principal components.

Figures 1 through 3 show the processes the model went through for each set of data. For all three figures, D(X) means the inputs of the model had been experimentally designed, SYS means that the model was ran with the experimentally designed inputs, PCA means principal components analysis was accomplished, CV means variance reduction had been performed through the method of control variates. C is the set of controls, Y is the original set of outputs of the model, Z is the set of principal components, and $Z(\hat{\beta})$ and $Y(\hat{\beta})$ are variance reduced versions of Z and Y.

Case 1. The first set of of data was generated from an experimentally designed model. The data were the principal components of the outputs of the experimentally designed model. Figure 1 shows the experimental design applied on the model. The outputs of the model were then transformed into principal components.

Case 2. The second set of data were generated from the model on which experimental design and variance reduction have been applied. The data, again, were the principal components of the outputs of the experimentally designed, variance-controlled model. Figure 2 shows experimental design being performed on the model. The outputs and the control candidates were then used to find the variance-controlled equivalents of the original outputs. Principal components analysis was applied on the variance-controlled responses.

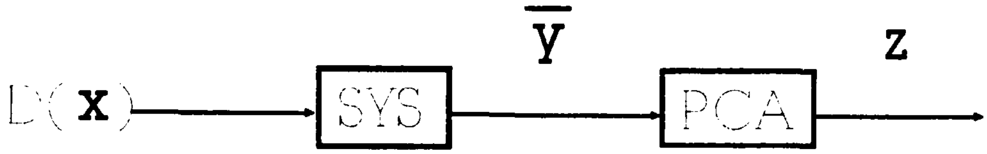

Figure 1. Case 1

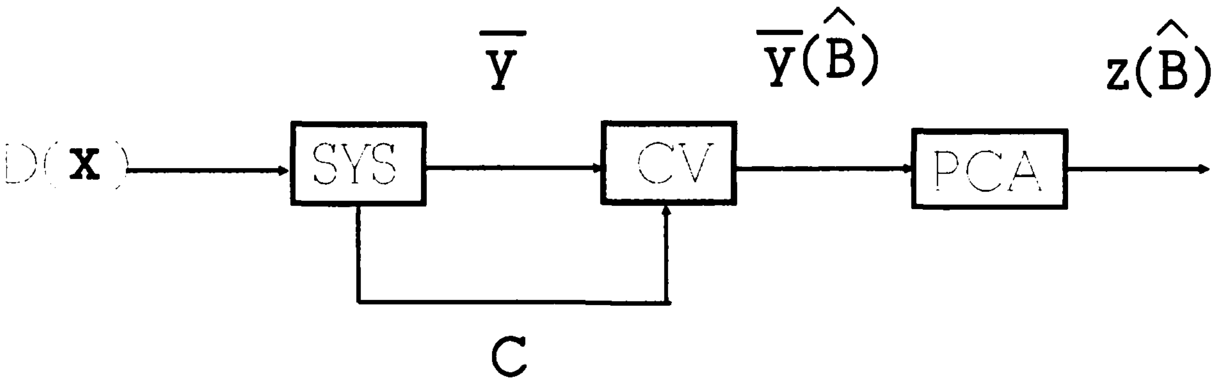

Figure 2. Case 2

Case 3. The third set of data were generated from an experimentally designed model whose outputs were first transformed into principal components before variance reduction was performed. Figure 3 shows experimental design applied on the model. The outputs were then transformed into principal components. Control variates was applied using principal components.

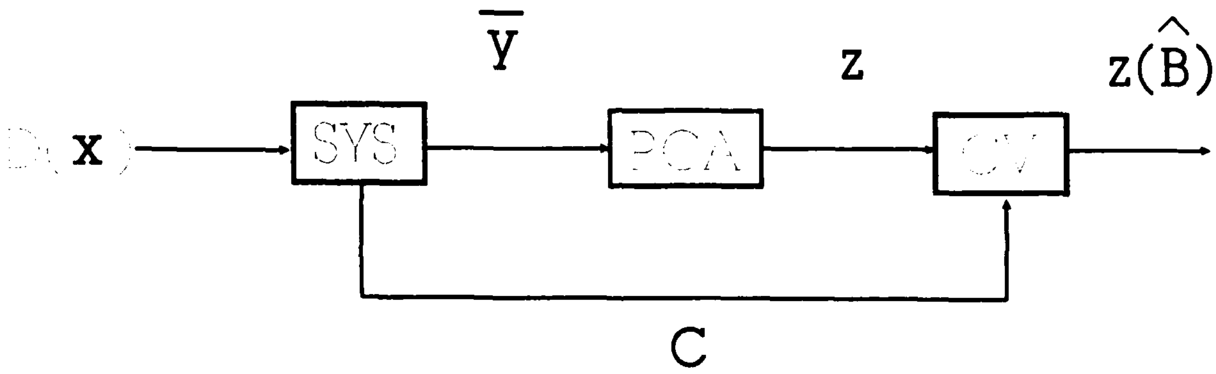

Figure 3. Case 3

II. Background and Literature Review

This chapter reviews literature relevant to the research proposal. Specifically, this research effort required a literature review of experimental design, control variates, and principal component analysis.

Experimental Design

Experimental designs have been developed since the 1930s, and a large amount of literature is available. However, most publications do not concentrate on simulation experiments. Most of the experimental design literature address techniques needed because of incomplete control over the experimental conditions. This is especially true in industrial and agricultural experiments. In simulation experiments, however, the analyst has full control over all of the factors. He simply makes changes in the inputs of the computer program. The only uncontrolled element is the pseudorandom number and, even then, the analyst can control the seed of that stream. Therefore, technical aspects such as randomization and blocking, which are discussed to a great extent in the experimental design literature are not great concerns in simulation designs (Kleijnen:260-261). Similarly, these concepts will not be discussed here.

The role of experimental design is to aid in ultimately finding a functional relationship between the inputs and outputs of a system of interest--ideally, an equation that would describe the system. One way of finding an equation for describing a system is usually to examine the outputs (or responses) of a system whose input variables have been set to all possible settings. In other words, by setting the system input variables to all possible combinations of factors and all possible levels, and then studying the system response to those combinations, an equation may be obtained that can adequately describe the behavior of

the system. However, to examine the system at each level and at each factor requires N computer simulation runs where N is

$$N = \Pi L_j$$

where factor j has L_j levels and there are k factors. N, therefore, can be a large number even for small values of j and k. The cost and time associated with these computer runs are usually the limiting factors in an experiment.

An experimental design (or a design of experiments) is used to restrict the number of factor-level combinations (or design points) that must be examined while retaining the capability to have sufficient data so as to adequately describe the behavior of the entire system. The objective of a design of experiments is to find a subset of combinations that can represent all the significant effects of all the combinations.

In an experimental design, the factor-level combinations or design points to be run are consolidated in a matrix called the design matrix. If the design matrix has orthogonal columns, the effects of factors can be assessed individually and independently. Further, including or excluding a factor does not change the estimates of the parameters associated with the other factors. (For an extensive description of the properties or advantages of orthogonality, see Box and Draper, 1987:76-77 or Neter et al. 1985:691-692.)

After a design matrix is chosen, a model can be formed in which the response or response vector is given as some function of the input variables and a parameter set associated with the input variables. The model can be written in the following form

$$y = f(x, \beta) + \varepsilon$$

where y is the vector of responses, x is the vector of inputs, β is the parameter set, and ε is the experimental error (which is assumed to be normally distributed with zero mean). Once a functional form, f, has been found, the least-squares criterion is used to decide which values for β are optimal. (For a complete explanation of least-squares criterion, consult Box and Draper, 1987:34-57 or Neter et al, 1985:210,238.)

There are three stages in experimental design. The first is a preliminary investigation of the factors--in this stage, the analyst identifies or screens the factors or factor interactions that have the greatest influence upon the the response. In the second stage, the analyst further investigates the important factors to actually select the factors and levels (the design points) to be included in the experiment. The final stage of an experimental design is to determine whether the model obtained by the experimental runs is adequate to explain the system (Kleijnen, 1974:265).

Preliminary Investigation. Designs that are useful in the first stage of experimental designs include two-level factorial designs. (Descriptions of these designs may be found in Box and Draper, 1987.) The computer simulation is run with this design matrix as the input. The output of the simulation is the vector of system responses where each response is associated with a design point. A regression is performed on the design matrix and vector of system response to obtain the β values or parameter set associated with the input variables.

As previously mentioned, the object of this first phase is to discover which effects are significant. A common method for checking for significance of effects is by plotting the parameters associated with the input variables (obtained through the first stage designs) on normal probability paper. If the plot of an effect falls out of a straight line, the analyst can conclude that the effect is

significant and should be investigated further (Bauer,1989:15-20). (For a detailed explanation of the technique and theory behind normal probability plots, see Box and Draper,1987:128-134)

As a point of interest, another supposedly common technique (according to Benski:174) is an approach where the analyst provides an "a priori" probability of an effect being significant and then calculates, using actual experimental results, the "a posteriori" probability of the significance of the effect. Benski, however, offers yet another alternative to the above two methods. His method combines a test for normality with a test for outliers. The method is based on the same assumptions as the above mentioned techniques but does not require the analyst to make subjective assessments concerning either the departure from a straight line or a priori probabilities.

Second Stage. Once the important factors and/or factor interactions have been discovered, the usual design used in the second stage of an experimental design is a full factorial design on the important factors or variables. In a full factorial design, all the factors and levels are examined. The result of the second stage design is, again, a vector of system responses.

A regression is again performed on the full factorial design matrix and the vector of system responses. The β values or parameter set associated with the input variables are obtained. These β values are the coefficients of the associated input variables in the model that describes the system.

Adequacy of the Model. The model obtained from the second stage is then checked for lack-of-fit. By comparing the actual responses of the system to the predicted responses of the model, a determination can be made as to the adequacy or aptness of the model. Several methods can be used to perform lack-of-fit test. These include the examination of Mallow's C_p statistic, R^2 and

adjusted-R^2 statistics, residual analysis, or F-tests. Box and Draper (1987,70-74) or Neter et al (1985,109-134) discuss various approaches.

Control Variates

The concept behind the theory of control variates is the selection of model input variables with known means and high correlations with the model response variables. The correlations of the inputs and the responses can lead to reductions in the variances of the responses

Univariate Simulation Response with a Single Control. Let Y be an estimator of μ_y, where μ_y is an estimator of a response of interest. Let C be another random variable with known mean μ_C and highly correlated with the response. The variable C is the control variable. The controlled estimator is given by

$$Y(b) = Y - b(C - \mu_C) \qquad (2.2.1)$$

where b is a constant called the control coefficient.

The variance of Y(b) is given by

$$Var[Y(b)] = Var(Y) + b^2 Var(C) - 2bCov(Y, C) \qquad (2.2.2)$$

A variance reduction will be realized if

$$2bCov(Y, C) > b^2 Var(C) \qquad (2.2.3)$$

The controlled estimator, Y(b), will have a smaller variance than the uncontrolled estimator, Y, if equation (2.2.3) holds. Calculus reveals that Y(b) has minimum variance when b is set equal to the optimal control coefficient given by

$$\beta = \frac{Cov(Y, C)}{Var(C)} \qquad (2.2.4)$$

Substituting (2.2.4) into (2.2.1) yields the optimal controlled estimator $Y(\beta)$

$$Y(\beta) = Y - [\frac{Cov(Y, C)}{Var(C)}](C - \mu_C) \qquad (2.2.5)$$

with the minimum variance given by

$$Var[Y(\beta)] = (1 - \rho_{YC}^2) Var(Y) \qquad (2.2.6)$$

where ρ_{YC} is the correlation coefficient between Y and C. Since the correlation coefficient in (2.2.6) is a squared term, the sign of the correlation does not matter; only the magnitude does. Thus, the higher the correlation, the higher the variance reduction.

The average of the uncontrolled observations Y is an unbiased point estimator of μ_Y. The average of the controlled observations $Y_i(\beta)$ is also an unbiased estimator of μ_Y. This is given by

$$\overline{Y}(\beta) = (\frac{1}{K}) \sum_{i=1}^{K} Y_i(\beta) \qquad (2.2.7)$$

where K is the sample size and

$$Y_i(\beta) = Y_i - \beta(C_i - \mu_C) \qquad (2.2.8)$$

In practice, Cov(Y,C) and Var(C) are unknown. Therefore, β is unknown and must be estimated. Bauer (1987a:6) gives an intuitive approach to estimating β by replacing the right-hand side of (2.2.4) with the appropriate sample statistics. This yields the least-squares solution. Under the assumption of joint normality between Y and C, the least squares solution is also the maximum likelihood solution. β is estimated by

$$\hat{\beta} = \frac{\sum_{i=1}^{K} (Y_i - \bar{Y})(C_i - \bar{C})}{\sum_{i=1}^{K} (C_i - \bar{C})^2} \qquad (2.2.9)$$

where

$$\bar{Y} = \sum_{i=1}^{K} \frac{Y_i}{K} \qquad (2.2.10)$$

and

$$\bar{C} = \sum_{i=1}^{K} \frac{C_i}{K} \qquad (2.2.11)$$

The point estimate of μ_y is given by

$$\bar{Y}(\hat{\beta}) = \sum_{i=1}^{K} \frac{Y_i(\hat{\beta})}{K} \qquad (2.2.12)$$

or

$$\bar{Y}(\hat{\beta}) = \bar{Y} - \hat{\beta}(\bar{C} - \mu_C) \qquad (2.2.13)$$

The variance of the point estimator is given by

$$\hat{Var}[\bar{Y}(\hat{\beta})] = \frac{\hat{Var}[Y(\hat{\beta})]}{K} \qquad (2.2.14)$$

where

$$\hat{Var}[Y(\hat{\beta})] = (1 - \hat{\rho}^2_{YC}) \hat{Var}(Y) \qquad (2.2.15)$$

Bauer (1987a:8-9) also provides the derivation of the interval estimate under the assumption of joint normality between Y and C.

The 100(1-α)% confidence interval on the point estimator for μ_Y is given by

$$\bar{Y}(\hat{\beta}) \pm t_{k-2}(1-\alpha/2)\{\hat{Var}[\bar{Y}(\hat{\beta})] s_{11}\}^{1/2} \qquad (2.2.16)$$

where

$$s_{11} = \frac{\sum_{i=1}^{K} (C_i - \mu_C)^2}{K \sum_{i=1}^{K} (C_i - \bar{C})^2} \qquad (2.2.17)$$

and $t_{k-2}(1-\alpha/2)$ is the $100(1-\alpha/2)$ percentile of Student's t-distribution with (k-2) degrees of freedom.

Since β is estimated, a smaller variance reduction will be realized than if β is known. The loss is quantified by what is named the loss factor (LF). The LF is defined as the ratio of the variance of the estimator of μ_Y, when the optimal β is not known, to the variance of the estimator when β is known. Bauer (1987a:9-10) provides the derivation of the loss factor which yields

$$LF = \frac{(K-2)}{(K-Q-2)} \qquad (2.2.18)$$

where Q is the number of controls and K is the number of replications.

This loss factor acts as a multiplier to the minimum variance ratio (MVR) given by

$$MVR = \frac{Var[\bar{Y}(\beta)]}{Var(\bar{Y})} \qquad (2.2.19)$$

The MVR represents the possible variance reduction when the optimal control coefficient is known. Multiplying (2.2.18) with (2.2.19) yields the variance ratio (VR). The VR represents the possible variance reduction when β is not known (Bauer, 1987a:9-10).

Univariate Simulation Response with Multiple Controls. Let Y be the univariate response with variance σ_Y^2, $\mathbf{C}$ be the (QX1) vector of controls, $\boldsymbol{\sigma}_{CY}$ be the (QX1) vector of covariances between Y and $\mathbf{C}$, and $\boldsymbol{\Sigma}_C$ be the (QXQ) covariance matrix of the controls. Then, (2.2.13) with multiple controls is given by

$$\bar{Y}(\hat{\boldsymbol{\beta}}) = \bar{Y} - \hat{\boldsymbol{\beta}}^T(\bar{\mathbf{C}} - \boldsymbol{\mu}_C) \qquad (2.2.20)$$

where $\hat{\boldsymbol{\beta}}$, $\bar{\mathbf{C}}$, and $\boldsymbol{\mu}_C$ are (QX1) vectors. The vector of optimal control coefficients, is then given by

$$\boldsymbol{\beta} = \boldsymbol{\Sigma}_C^{-1}\boldsymbol{\sigma}_{CY} \qquad (2.2.21)$$

Since the covariance matrices are usually unknown, $\boldsymbol{\beta}$ can be estimated by substituting the sample analogs of $\boldsymbol{\Sigma}_C$ and $\boldsymbol{\sigma}_{CY}$ into (2.2.21). This leads to the following equation

$$\hat{\beta} = S_C^{-1} S_{CY} \qquad (2.2.22)$$

where S_C^{-1} is the inverse of the (QXQ) sample covariance matrix of the controls, and S_{CY} is the (QX1) vector of sample covariances between the univariate response and the vector of controls (Bauer:1987a:12-13).

Under the assumption of joint normality of Y and C, $\bar{Y}(\hat{\beta})$ is unbiased for μ_Y and an exact $100(1-\alpha)\%$ confidence interval is given by

$$\bar{Y}(\hat{\beta}) \pm t_{K-Q-1}(1-\alpha/2)\, D\, S_{Y|C} \qquad (2.2.23)$$

where

$$D^2 = K^{-1} + (K-1)^{-1}(\bar{C}-\mu_C)^T S_C^{-1}(\bar{C}-\mu_C) \qquad (2.2.24)$$

$$S_{Y|C}^2 = (K-Q-1)^{-1}(K-1)(S_Y^2 - S_{CY}^T S_C^{-1} S_{CY}) \qquad (2.2.25)$$

$t_{K-Q-1}(1-\alpha/2)$ is the $100(1-\alpha/2)$ percentile of Student's t-distribution with (K-Q-1) degrees of freedom, and S_Y^2 is the sample variance of Y. Experimental results have shown that the assumption of joint multivariate normality is robust (Bauer et al, 1988:2-5).

Multiple Simulation Responses with Multiple Controls. Bauer et al (1987b:1-3) provide an outline of the theoretical formulas for the case when there are P response variables and Q control variables. In terms of the notation, the univariate response Y becomes a (PX1) vector of response variables, β becomes a (PXQ) matrix of control coefficients, and the scalar deviation S_Y becomes the sample covariance matrix of the response vector. Under the

assumption that Y and C have the joint multivariate normal distribution, $\bar{Y}(\hat{\beta})$ is an unbiased estimator of μ_Y, and an exact 100(1- α)% confidence ellipsoid for μ_Y is given by

$$[\bar{Y}(\hat{\beta})-\mu_Y]^T S_{Y|C}^{-1}[\bar{Y}(\hat{\beta})-\mu_Y] < P(K-Q-1)(K-P-Q)^{-1}D\, F(1-\alpha;P,K-P-Q) \qquad (2.2.26)$$

where D^2 and $S_{Y\,C}{}^2$ are as in (2.2.24) and (2.2.25) and $F(1-\alpha;\, m_1, m_2)$ is the $100(1-\alpha)$ percentile of the F-distribution with m_1 and m_2 degrees of freedom (Bauer et al, 1987b:2).

Principal Components Analysis

The objective of principal components analysis (PCA) is to examine the interdependence structure of a set of variables. Some specific objectives include data reduction, interpretation, and testing of prior hypothesis. For the purposes of this research, the variables being studied are the response variables of a system of interest and the specific objectives are data reduction and, to a lesser extent, interpretation. (The system of interest is the same system on which experimental design was performed.)

The goal of PCA is to reduce p number of original variables to q factors with q being much less than p. The characteristics of the q factors may be more readily interpreted than the characteristics of the original p variables. For example, let p variables represent a student's scores in several tests. Each score by itself does not really provide insight. If the scores were somehow compiled into two ratings (the q factors) that were linear combinations of the p scores, the factors could, for example, represent the student's math and verbal aptitudes.

Let X be a p-dimensional random variable with mean μ and covariance matrix Σ. The concept behind principal components analysis is to find a new set of variables, Y, which are uncorrelated and whose variances decrease--the first variable would contain the largest variance while the last variable would have the least (Dillon 24-45). Each Y_j is taken to be a linear combination of the X's so that

$$\begin{aligned} Y_j &= a_{1j}X_1 + a_{2j}X_2 + \ldots + a_{pj}X_p \qquad (2.3.1) \\ &= a_j^T X \end{aligned}$$

where a^T_j is a vector of constants. Equation (2.3.1) contains an arbitrary scale factor. Therefore, the condition that $a^T_j a_j = \sum_{k=1}^{p} a_{kj}^2 = 1$ is imposed. This normalization procedure ensures that a unique solution is obtained.

The first principal component, Y_1, is found by choosing a_1 so that Y_1 has the largest possible variance. In other words, a_1 is chosen so as to maximize the variance of $a^T_1 X$ subject to the constraint $a^T_1 a_1 = 1$.

The second principal component is found by choosing a_2 so that Y_2 has the largest possible variance for all combinations of the form (2.3.1) which are uncorrelated with Y_1. Similarly, $Y_3, \ldots, Y_p$ are chosen so as to be uncorrelated with each other and to have decreasing variance.

Beginning with finding the first component, a_1 is chosen so as to maximize the variance of Y_1 subject to the normalization constraint, $a^T_1 a_1 = 1$. The variance of Y_1 is given by

$$\begin{aligned} Var(Y_1) &= Var(a_1^T X) \qquad (2.3.2) \\ &= a_1^T \Sigma a_1 \end{aligned}$$

Thus, $a_1^T \Sigma a_1$ is the objective function to be maximized.

An accepted procedure for maximizing a function of several variables subject to one or more constraints is the method of Lagrange multipliers. This method uses the result from the calculus at a stationary point. The partial derivatives of a function subject to a constraint all vanish. For example, given a function of p variables, $f(x_1, ..., x_p)$, subject to a constraint $g(x_1, ..., x_p)$, there exists a scalar called the *Lagrange multiplier* such that

$$\frac{\delta f}{\delta x_i} - \lambda \frac{\delta g}{\delta x_i} = 0, \quad i=1, ..., p \qquad (2.2.3)$$

at any stationary point. These p equations, together with the constraint, are enough to determine the coordinates of the stationary points. Further investigation is needed to see if a stationary point is a maximum, minimum, or saddle point. It is helpful to form a new function, $L(x)$, such that

$$L(\mathbf{x}) = f(x) - \lambda [g(\mathbf{x}) - c] \qquad (2.3.4)$$

where the term in the square brackets, $[g(\mathbf{x}) - c]$, is zero. Then the set of equations in (2.3.3) may be written as

$$\frac{\delta L}{\delta \mathbf{x}} = 0 \qquad (2.3.5)$$

Applying the Lagrange multiplier method to (2.3.2) with the normalization constraint yields

$$L(a_1) = a_1^T \Sigma a_1 - \lambda (a_1^T a_1 - 1) \qquad (2.3.6)$$

This leads to

$$\frac{\delta L}{\delta a_1} = 2 \Sigma a_1 - 2 \lambda a_1 \qquad (2.3.7)$$

Setting (2.3.7) equal to 0 yields

$$(\Sigma - \lambda I) a_1 = 0 \qquad (2.3.8)$$

The identity matrix I is inserted into (2.3.8) so that the term in brackets is of the correct order, (pXp). If (2.3.8) is to have a solution for a_1 that is other than the null vector, then $(\Sigma - \lambda I)$ must be a singular matrix. Thus λ must be chosen so that

$$\left| \Sigma - \lambda I \right| = 0 \qquad (2.3.9)$$

Thus a non-zero solution for (2.3.8) exists if and only if λ is an eigenvalue of Σ. But Σ will generally have p eigenvalues, which must all be nonnegative as Σ is a positive definite matrix. If the eigenvalues are denoted by $\lambda_1, \lambda_2, \ldots, \lambda_p$ and assuming they are distinct, the eigenvalue that will determine the first principal component is the largest eigenvalue, λ_1. Then using (2.3.8), the principal component, a_1, is the eigenvector of Σ corresponding to the largest eigenvalue (Chatfield, 1980:59-60).

The second principal component, $Y_2 = a^T{}_2X$, is obtained by an extension of the above argument. In addition to the scaling constraint that $a^T{}_2 a_2 = 1$, a second constraint exists that Y_2 should be uncorrelated with Y_1.

The covariance between Y_1 and Y_2 is given by

$$\begin{aligned} Cov(Y_1, Y_2) &= Cov(a^T{}_2X, a^T{}_1X) \qquad (2.3.10) \\ &= E[\,a^T{}_2(X-\mu)(X-\mu)^T a_1\,] \\ &= a^T{}_2 \Sigma\, a_1 \end{aligned}$$

The covariance is required to be zero if Y_1 and Y_2 are to be uncorrelated. But since $\Sigma a_1 = \lambda_1 a_1$, an equivalent condition is that $a^T{}_2\, a_1 = 0$. In other words, a_1 and a_2 should be orthogonal.

In order to maximize the variance of Y_2, $a^T{}_2 \Sigma\, a_2$, subject to the two constraints, two Lagrange multipliers are introduced. The two multipliers are denoted by λ and δ respectively. The function for maximizing the variance of Y_2 is given by

$$L(a_2) = a_2^T \Sigma a_2 - \lambda(a_2^T a_2 - 1) - \delta a_2^T a_1 \qquad (2.3.11)$$

At the stationary points

$$\frac{\delta L}{\delta a_2} = 2(\Sigma - \lambda I)a_2 - \delta a_1 = 0 \qquad (2.3.12)$$

is required. This equation premultiplied by $a^T{}_1$ gives

$$2a_1^T \Sigma a_2 - \delta = 0 \qquad (2.3.13)$$

since $a^T_1 a_2 = 0$. But from (2.3.10), $a^T_1 \Sigma a_2$ is required to be zero, so that δ is zero at the stationary points. Thus (2.3.12) becomes

$$(\Sigma - \lambda I) a_2 = 0 \qquad (2.3.14)$$

This time λ is chosen to be the second largest eigenvalue of Σ, and a_2 to be the corresponding eigenvector.

Continuing this argument, the i_{th} principal componentresults in the eigenvector associated with the j_{th} largest eigenvalue. More specifically, the normalized eigenvectors of Σ are used to form the principal components of **X**. The principal components are uncorrelated and the variance of the i_{th} principal component is λ_i (Chatfield, 1980:60).

There is no difficulty in extending the above argument to the case where some of the eigenvalues of Σ are equal. In this case there is no unique way for choosing the corresponding eigenvectors, but as long as the eigenvectors associated with multiple roots are chosen to be orthogonal, the argument carries through.

An important result from principal components analysis is that the sum of the variances of the original variables is equal to the sum of the variances of the principal components. It is also possible to state that the i^{th} principal component accounts for a proportion $\lambda_i \Big/ \sum_{i=1}^{p} \lambda_i$ of the total variance of the original data. This type of statement may also be made about the first m

components; that the first m components account for a proportion $\sum_{i=1}^{m}\lambda_i \Big/ \sum_{i=1}^{p}\lambda_i$ of the total variance (Chatfield,1980:61).

Using the Correlation Matrix. An alternative way of calculating principal components is by transforming the original set of variables after they have been standardized to have unit variance. This essentially means that the principal components are obtained from the correlation matrix P instead of from the covariance matrix Σ. The mathematical derivation is exactly the same--the eigenvectors are eigenvectors of P. However, the eigenvectors of P are not the same as the eigenvectors of Σ. By analyzing P rather than Σ, the analyst has decided to make the variables equally important--the contribution to the total variance is equal for all the variables (Bauer,1990:27-28).

For the correlation matrix, the diagonal terms are all one. The original variables contribute exactly the same variance before principal components analysis is performed. Thus the trace (the sum of the diagonal terms of a square matrix) of the correlation matrix is equal to p where p is the number of rows of P. The sum of the eigenvalues of P will also be equal to p. Therefore, the proportion of the total variance accounted for by the j^{th} component is λ_j/p (Collins:62).

Component Loadings. The matrix of component loadings is a matrix that shows the correlations between the original variables and the principal components.

Summary

This chapter has reviewed the literature pertinent to this research. An experimental design was needed to produce the outputs on which control

variates and principal components analysis were applied. The next chapter, Chapter 3, discusses the actual work that was performed to accomplish the objective as stated in the first chapter. Chapter 4 will present the results and Chapter 5 will submit the conclusions and recommendations.

III. Methodology

To achieve the research objective, the concepts discussed in the literature review had to be examined in great detail. In addition, a model had to be selected and a computer program was used. This chapter discusses the model used in the research, the experimental design employed to identify the significant factors, and the steps that were taken to produce the data specified for the three cases.

Model. The model chosen for this research is representative of a class of queueing systems which are frequently analyzed in computer performance modeling. This system has been studied extensively and workable control variables have been developed by several authors.

The simulation model selected for study in this research is a model of the network portrayed in Figure 4. Node 1 has *N* servers, where *N* is the finite number of customers of all types. This node might be a room filled with N interactive computer terminals. The node labeled 2 might be a holding area or buffer which has a capacity less than the number of terminals. The nodes labeled 3 through S are single server queues with the customers being served in order of arrival. Node 3 might be a central processing unit (CPU) with nodes 3 through S as peripheral devices accessed by the CPU.

The specific model studied in this research has a total of seven nodes--the computer room, the holding area or the buffer, the CPU, and four peripheral devices. There are 25 terminals in the computer room and the buffer has a capacity of five (Bauer,1987a:84-90).

The S by S transition matrix that characterizes the flow of customers in the network has the form

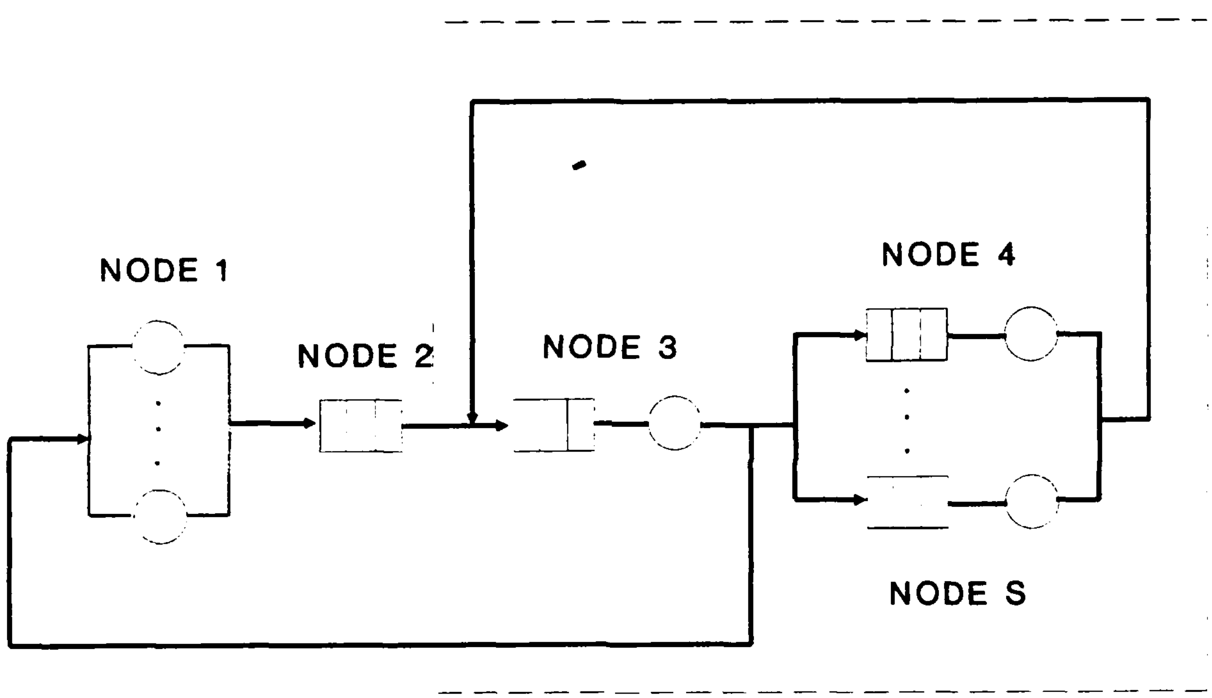

Figure 4. Network Model

$$P(d) = \begin{bmatrix} 0 & 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 \\ p_1(d) & 0 & 0 & p_4(d) & \dots & p_S(d) \\ 0 & 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 \end{bmatrix}$$

where $p_k(d)$, $k = 1, \dots, S$, is the one-step transition probability (for a customer of type d) from node 3, the CPU, to the remaining nodes. Table 1 shows the actual probabilities used in the S by S transition matrix.

Table 1. Transition Probabilities From Node 3 to Node i

Node	Probability
1	0.2
2	0
3	0
4	0.36
5	0.36
6	0.04
7	0.04

In this network, an assumption has been made that every customer that requests service from the CPU is immediately granted access. (However, only five customers can gain entry to the holding area.) Other assumptions about the network are as follows (Bauer,1987a:66):

1) The network has Markovian routing--the next node visited depends only on the current location.

2) the service times for the j-th type of customer at the i-th service node are drawn independently from a given probability distribution, F_{ij} (*), with finite mean and variance. There is no service time associated with node 2, the buffer or holding area.

3) Service time sequences and sequences of nodes visited are mutually independent.

An interesting feature that was deliberately built into the model was that nodes 4 through 7 contribute equal "delays" in a customer's time in system. This delay value of one time unit is found by multiplying the steady-state probability of being in nodes 4 through 7 by the respective node mean service time.

A listing of the simulation program for this model is given in Appendix 1.

Experimental Design. In order to execute the research as outlined in the cases described in Chapter 1, Introduction, model outputs were generated. The outputs generated had to be representative of all the possible outputs the model can produce. Thus, an experimental design was implemented on the model to determine which of the model parameters are significant. Once these significant parameters were found, another design was then used to produce the outputs that were used for further study in Cases 1, 2, and 3. By implementing an experimental design on the model, the outputs studied were representative of the model outputs as a whole.

The controllable inputs of the model are the service times at each node. The type of distribution, its mean, and variance describe the service times of each node. The response variable examined during the experimental design portion of the research is the customer's mean time in system.

As was mentioned in the literature review, a preliminary evaluation of a model is needed to determine or screen the important factors. A 2^7 design was used in this preliminary phase. (The mean service time was ignored at this phase to avoid having to simulate 3^7 design points.) This means that a full factorial was done on a percentage deviation from the mean. The data initially used was ± 10% of the mean service time. However, plotting the effects on a normal probability plot showed no effects exhibiting significance at the 10% deviation. Changing the deviation level to ± 20% led to finding significant effects of factors 4, 5, 6, and 7--service times for nodes 4, 5, 6, and 7 are the significant factors. Figure 5 shows the normal probability plot of the β values obtained from regressing the full factorial design matrix on the output (mean time in system).

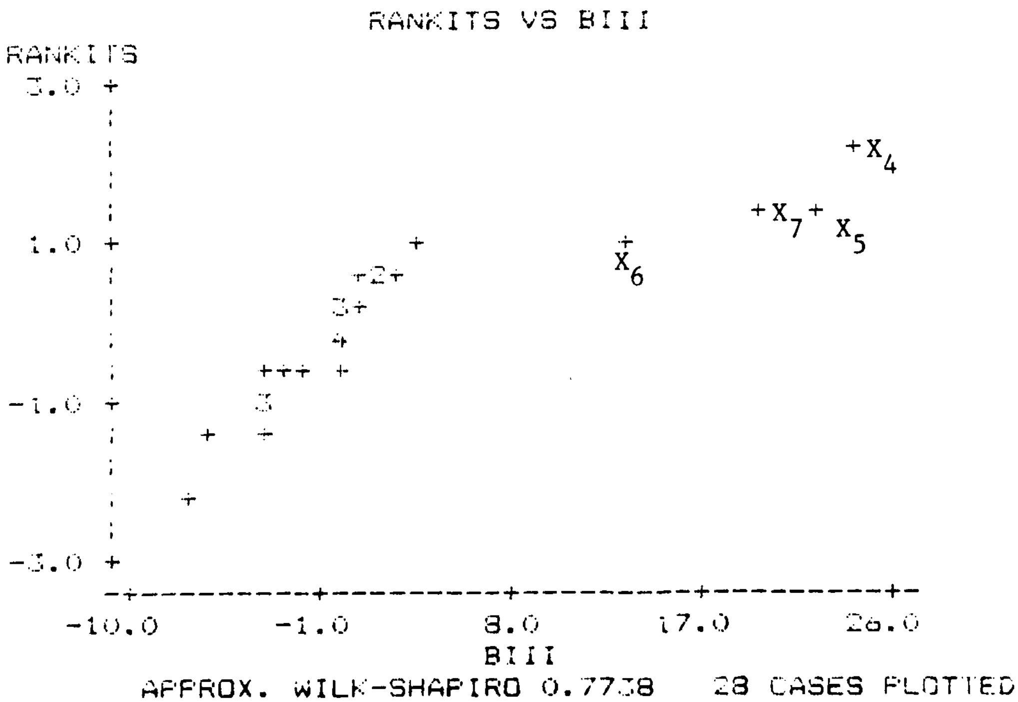

Figure 5. Normal Probability Plot of the β values

A full factorial design of the significant main effects was run. Another regression was performed on the system response and model inputs to obtain the final model. (The SAS software system was used to perform the regressions. The SAS runs used to accomplish the first and second stage of the experimental design are found in Appendix 2.)

Case 1. The next step after the experimental design was to execute runs of the model to obtain principal components of the outputs. The output vector that was studied for the three cases contains [T, U(1), U(3), U(4), U(5), U(6), U(7)] where T is mean time in system and U(j) is the utilization rate at node j. The model was run for 5000 time units with the statistics cleared after the first 2000 time units.

Twenty simulation runs were executed at each of the design points of the design matrix selected after the experimental design analysis. The design selected was a full factorial design on the four significant factors--service times for nodes 4, 5, 6, and 7--while keeping the non-significant factors (service times for nodes 1, 2, and 3) at their mean levels. An additional set of runs was made with all the node service times set at their mean levels--this set of runs is labelled as Design point 17. The design matrix is shown in Table 2.

X_j is service time at node j--the value "-1" means the node service time was set to 20% below its mean, the value "1" means the the node service time was set to 20% above its mean, and the value "0" means the node service time was set to its mean value. The column labelled "Run" shows the alphabetic notation for each design point. A special notation of "1" is used to represent the set of runs when all the significant factors were at their low levels. A notation of "0" is used to represent the design point in which all the node service times were at their mean levels.

The mean of each response of interest was obtained. A 17 by 7 data matrix resulted--seventeen means each for seven responses A principal components

analysis was performed on the data. Table 3 shows the original data matrix--T stands for time in system and U(j) stands for utilization rate at node j. Nodes 1 and 2 have zero utilization rates. The principal components were obtained through use of the STATISTIX software.

Table 2. Experimental Design Matrix

Design point	X1	X2	X3	X4	X5	X6	X7	Run
1	0	0	0	-1	-1	-1	-1	1
2	0	0	0	1	-1	-1	-1	d
3	0	0	0	-1	1	-1	-1	e
4	0	0	0	1	1	-1	-1	de
5	0	0	0	-1	-1	1	-1	f
6	0	0	0	1	-1	1	-1	df
7	0	0	0	-1	1	1	-1	ef
8	0	0	0	1	1	1	-1	def
9	0	0	0	-1	-1	-1	1	g
10	0	0	0	1	-1	-1	1	dg
11	0	0	0	-1	1	-1	1	eg
12	0	0	0	1	1	-1	1	deg
13	0	0	0	-1	-1	1	1	fg
14	0	0	0	1	-1	1	1	dfg
15	0	0	0	-1	1	1	1	efg
16	0	0	0	1	1	1	1	defg
17	0	0	0	0	0	0	0	0

<u>Case 2.</u> For this case, variance reduction was performed before principal components of the outputs were found.

The candidate control variables selected can be classified into three basic types: 1) service time variables, 2) flow variables, and 3) work variables. All of these variables were collected at each node for each customer type. Service time variables are the sample mean service times. Flow variables are the

sample proportion of departures from particular nodes relative to the total number of departures from all nodes. Work variables are the product of the service time variables and the flow variables.

Table 3. Data Used in Case 1

Design point	T	U3	U4	U5	U6	U7
1	154.22	0.4741	0.3843	0.3816	0.3737	0.3830
2	205.01	0.4016	0.4848	0.3260	0.3295	0.3260
3	201.34	0.4140	0.3313	0.4956	0.3314	0.3253
4	220.47	0.3779	0.4548	0.4548	0.2919	0.2926
5	213.92	0.3963	0.3210	0.3146	0.4729	0.3150
6	246.45	0.3576	0.4362	0.2893	0.4381	0.2991
7	246.16	0.3627	0.2907	0.4363	0.4650	0.2849
8	272.26	0.3283	0.3927	0.3886	0.4103	0.2659
9	214.45	0.3952	0.3142	0.3186	0.3201	0.5084
10	248.83	0.3620	0.4212	0.2791	0.2811	0.4612
11	234.07	0.3630	0.2926	0.4405	0.2976	0.4294
12	258.86	0.3361	0.4029	0.4059	0.2562	0.4041
13	229.14	0.3694	0.2945	0.3039	0.4383	0.4244
14	261.30	0.3459	0.4185	0.2746	0.4056	0.3825
15	273.79	0.3318	0.2674	0.3986	0.4162	0.3991
16	304.60	0.3054	0.3727	0.3591	0.3789	0.3680
17	234.65	0.3684	0.3719	0.3721	0.3597	0.3844

The work variables may be standardized to have a multivariate normal distribution with a zero mean vector and an identity covariance matrix. This is done by defining the work variables as

$$X_k^*(t) = \frac{\sqrt{f(k,t)}}{w_k(f(t))} \sum_{i=1}^{f(k,t)} \frac{U_i(k) - \mu_k}{\sigma_k}$$

where w_k = relative frequency with which a customer visits station k, and $f(k,t)$ = number of service times that are finished at station k during the time period (0,t) (Lavenberg et al, 1982:182-202).

The moments of the flow variables are unknown in general. Hence, standardization of these variables cannot be accomplished, and these candidate control variables are discarded in favor of a standardized multinomial control. These new controls are called "routing variables".

All routing in the selected model is done from the CPU. Define an indicator variable on the event of the l^{th} departure from the CPU to station j

$$U_j(l) = 1 \text{ if the } l^{th} \text{ departing customer goes to station } j$$
$$= 0 \text{ otherwise}$$

From the description of the model, $p_j(*)$ was defined to be the probability of transition from the CPU to station j. N(t) is the total number of transitions from the CPU up to time t. A standardized equation for routing control variables is then

$$X_j^R(t) = \sum_{l=1}^{N(t)} \frac{U_j(l) - p_j(*)}{\sqrt{N(t)(1-p_j(*))p_j(*)}} \quad \text{for } j = 1, \ldots S \text{ if } N(t) > 0$$
$$= 0 \qquad\qquad \text{if } N(t) = 0$$

It can be shown that these standardized control variables also have the properties of zero mean vector and identity covariance matrix (Bauer, 1987a:95-97; Bauer, 1987b:4).

A computer program used in this research was POST.FOR (shown in Appendix 3). The program took model inputs of mean service times, probability of transition from the CPU to any of the other nodes, steady state probabilities of a customer being in a particular node, and the actual number of customers transitioning from the CPU to a particular node, and calculated the standardized work and routing variables. Once this calculation was done, control variates was accomplished more readily. Data that resulted from Case 1 runs were inputs directly for POST.FOR.

Recall from Chapter 2 the review on control variates. The equation used in control variates is given as

$$Y(\beta) = Y - \beta(X-\mu)$$

$Y(\beta)$ has a smaller variance than Y if correlation exists between the inputs and responses. The values of Y are obtained directly from the outputs of the model and the terms $(X-\mu)$ are the candidate controls--the standardized work and routing variables--with μ being a zero vector. The above equation can then rewritten as

$$Y = Y(\beta) + \beta(X-\mu)$$

After a regression is performed, values of the responses on which control variates has been performed is given directly by the β_0 values obtained from the regression. Therefore, variance-controlled values of the responses were obtained through regression on the observed Y and the control candidates. Appendix 4 contains the SAS program that performed this regression.

Once the variance-controlled responses were obtained, a data matrix similar to the 17 by 7 matrix in Case 1 was constructed. This time the contents were seven variance-controlled responses for each design point. The data matrix is shown in Table 4. TB stands for variance-controlled time in system and U(j)B stands for variance-controlled utilization rate for node j. Principal components analysis was performed using this data matrix.

Table 4. Data Used in Case 2

Design point	TB	U3B	U4B	U5B	U6B	U7B
1	155.17	0.4772	0.3791	0.3637	0.3766	0.3765
2	205.54	0.4011	0.4749	0.3237	0.3295	0.3144
3	201.34	0.4086	0.3297	0.4956	0.3235	0.3381
4	221.78	0.3818	0.4582	0.4528	0.2904	0.2939
5	206.39	0.3953	0.3251	0.3146	0.4728	0.3226
6	230.01	0.3775	0.4488	0.2981	0.4433	0.3009
7	238.28	0.3648	0.2950	0.4386	0.4476	0.2910
8	257.17	0.3418	0.4061	0.4114	0.3950	0.2672
9	210.59	0.3952	0.3152	0.3176	0.3129	0.4955
10	243.90	0.3561	0.4256	0.2866	0.2698	0.4394
11	234.57	0.3660	0.2907	0.4274	0.2983	0.4316
12	269.55	0.3287	0.3989	0.3991	0.2585	0.3953
13	244.35	0.3591	0.2854	0.2877	0.4326	0.4345
14	290.15	0.3151	0.3739	0.2578	0.3868	0.3816
15	261.65	0.3379	0.2702	0.4093	0.3981	0.4235
16	294.14	0.3098	0.3773	0.3637	0.3718	0.3633
17	234.71	0.3624	0.3692	0.3722	0.3601	0.3849

Case 3. Case 3 follows directly from Case 1 and used the same methodology as Case 2. Since the principal components were already found,

all that was needed was to run the control variates regression using the principal components found for Case 1, and the same input variable information used in Case 2.

The regression of the principal components found for Case 1 and the candidate control variables described in Case 2 resulted in values of β which were coefficients of the actual control variables. Appendix 5 contains the SAS program that performs this regression.

Recall again the control variates equation given by

$$Y(\beta) = Y - \beta(X - \mu)$$

To obtain the variance-controlled principal components of the model (which is the purpose of Case 3), the above equation was used. For this case, Y was actually the vector of principal components found in Case 1, the terms $(X - \mu)$ were the means of the candidate control variables of Case 2, and $Y(\beta)$ were the variance- controlled principal components. The variance-controlled principal components were obtained by subtracting from the original principal components the control variables multiplied by the corresponding β value found through the regression.

The next chapter discusses the analysis, comparison, and inference obtained from the experiments.

IV. Results and Observations

This chapter presents the results and observations gained after the experimentation, described in the previous chapter, was completed.

Model. The coded equation for the model obtained after a full factorial design on the significant effects was found to be

$$Y = 212.888 + 21.469X_4 + 14.306X_5 + 13.388X_6 + 8.212X_7 + 11.482X_{47} - 18.166X_{55}$$

where Y is the mean time in system and X_j is the mean service time of node j. An examination of the residual plots of the model showed aptness.

Case 1. The STATISTIX software package was used for principal components analysis. STATISTIX uses the data correlation matrix in computing principal components. A data correlation matrix has all ones on the diagonal. This can be interpreted as all original variables contributing the same amount of variance--namely, one--before principal components analysis is performed. Therefore, after principal components analysis has been performed, a common procedure for discarding principal components is to discard those principal components that have eigenvalues less than one. These discarded principal components have contributed less variance than any of the original variables.

Table 5 shows the STATISTIX output for Case 1. The table of eigenvalues showed four principal components with eigenvalues greater than one. These four principal components explained 99.4% of the total variance of the data.

The eigenvectors and loadings matrix also described interesting characteristics of the principal components.

1) Looking only at factor loadings with values greater than 0.5, principal component 1 was made up of time in system and the CPU utilization rate. The

time in system coefficient of the loadings matrix was of opposite sign from the utilization rate. This principal component may be considered a contrast of time in system and the CPU utilization rate. The description of the network model permits the following interpretation. If a customer is not in node 3, it is in one of the other nodes driving up the customer's time in system. Node 3 has a mean service time of 1 time unit. All the other node mean service times are greater than one time unit. If a customer is utilizing node 3, it is not raising the customer's time in system elsewhere.

Table 5. STATISTIX Output for Case 1

	Eigenvalues	% of Variance	Cumulative % of Variance
1	2.050	34.2	34.2
2	1.439	24.0	58.2
3	1.279	21.3	79.5
4	1.198	20.0	99.4
5	2.637E-02	0.4	99.9
6	7.260E-03	0.1	100.0

Vectors

Factor	1	2	3	4	5	6
T	-0.6825	0.0708	-0.0316	0.1573	0.3998	0.5862
U3	0.6736	-0.1139	0.0494	-0.1866	0.3598	0.6054
U4	0.0717	0.2440	-0.8320	-0.1058	0.4145	-0.2450
U5	0.1758	-0.2967	0.0891	0.8149	0.3897	-0.2392
U6	-0.2057	-0.6305	0.1669	-0.5005	0.4291	-0.3126
U7	0.0454	0.6610	0.5181	-0.1215	0.4504	-0.2737

2) The second principal component also had two factors with loading coefficients greater than 0.5--utilization rate of node 6 and utilization rate of node 7. The signs of the coefficients of the loadings matrix were opposite. Nodes 6 and 7 have equal mean service time and the probability of transition from the CPU to these nodes are also equal.

3) The third principal component again had two factors with coefficients greater than 0.5-- utilization rates of node 4 and node 7. The factors were also opposite in sign. Considering the description of the model allows the following explanation. A customer has a 0.36 probability of transitioning from the CPU to node 4 and node 4 has a mean service time of 2.78 time units. On the other hand, a customer has a 0.04 probability of transitioning from the CPU to node 7 but node 7 has a mean service time of 25.0 time units. Apparently node 4 takes away customers that could go to node 7 because customers have a higher probability of going to node 4. Node 7, however, has a higher mean service time and thus ties up customers that could go to node 4.

4) The fourth principal component had two factors with coefficients greater than 0.5--utilization rate for nodes 5 and 6. The interpretation of this principal component was similar to the interpretation of the third principal component.

Factor scores were obtained by multiplying the eigenvectors of the principal components by the original variables. Recall the equation,

$$
\begin{aligned}
Y_j &= a_{ij}X_1 + a_{2j}X_2 + \dots + a_{pj}X_p \\
&= \mathbf{a}_j^T X
\end{aligned}
$$

from the Chapter 2 review of principal components. Y_j is the factor score, $\mathbf{a}_j^T$ is the eigenvector, and **X** is the vector of original variables.

The factor scores of the four principal components were calculated for each design point and shown in Table 6. PC(j) stands for the j^{th} principal component.

A scatter plot of the first two principal component is shown in Figure 6. Each point has the alphabetic notation (described in Table 2) to indicate which design point it represents.

Table 6. Factor Scores of Case 1 Principal Components

Design Point	PC1	PC2	PC3	PC4
1	3.4697	-0.3731	0.1425	-0.7841
2	1.2868	0.4237	-1.9106	-0.6216
3	1.8444	-0.9808	0.3088	1.6190
4	0.9750	-0.1386	-1.7468	1.5305
5	0.3441	-1.5632	0.4368	-1.4741
6	-0.7873	-0.6696	-1.3644	-1.3675
7	-0.5614	-2.2781	0.6670	0.4702
8	-1.5218	-1.2036	-1.0672	0.4554
9	0.9338	1.7328	1.6393	-0.6198
10	-0.3834	2.4103	-0.3363	-0.5419
11	0.3085	0.6648	1.3488	1.4648
12	-0.5000	1.5152	-0.4616	1.4534
13	-0.2788	-0.1083	1.4744	-1.3093
14	-1.1851	0.5200	-0.6228	-1.2938
15	-1.5326	-0.4746	1.6118	0.4884
16	-2.4880	0.2843	-0.1892	0.4313
17	0.0761	0.2388	0.0696	0.0891

<u>Case 2.</u> The result of the regression to find the variance-controlled outputs are shown in Table 4. The values are labelled TB, U3B, U4B, U5B, U6B, and U7B for time in system and utilization rates of the node j, respectively, to indicate that control variates had been performed .

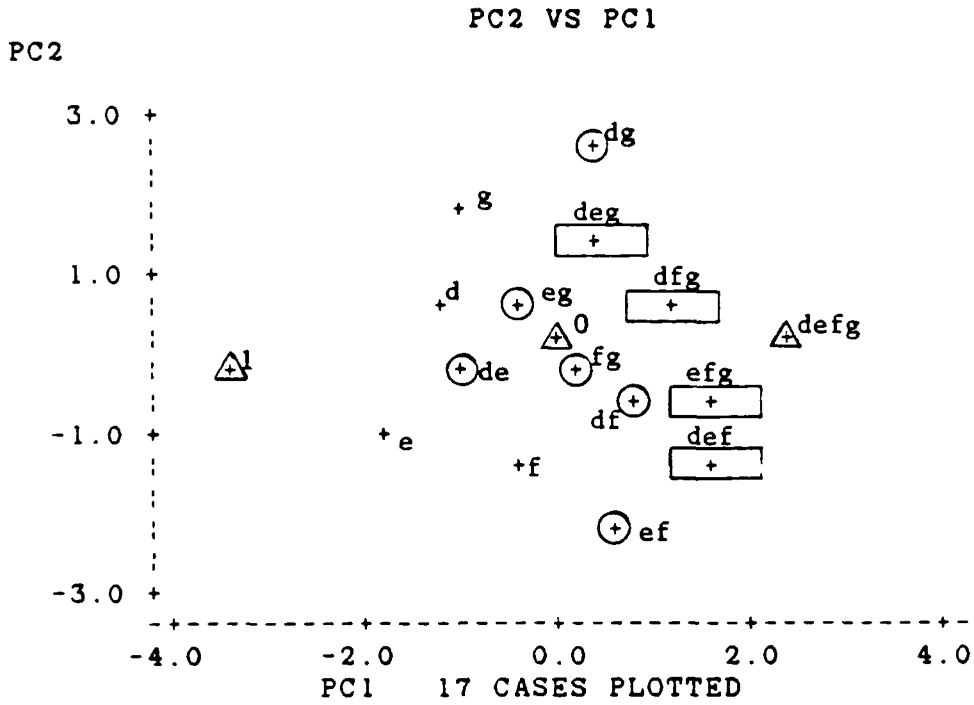

Figure 6. Scatter Plot of the First Two Principal Components

Again, principal components analysis was performed, this time on these variance-reduced outputs. The STATISTIX results are shown on Table 7.

The table of eigenvalues shows four principal component having eigenvalues greater than one. The four principal component accounts for 99.3% of the total variance.

The eigenvectors and loadings matrix also described interesting characteristics of the principal components.

1) Looking only at factors with coefficient values greater than 0.5, principal component 1 was made up of the variance-controled time in system and CPU utilization rate. The time in system coefficient of the loadings matrix was of

opposite sign from the utilization rate. This principal component may be considered a contrast of the variance-controlled time in system and the variance-controlled CPU utilization rate. The same explanation can be made about this factor pattern as the expalnation about the factor pattern of the first principal component of the original outputs.

Table 7. STATISTIX Output for Case 2

	Eigenvalues	% of Variance	Cumulative % of Variance
1	2.096	34.9	34.9
2	1.426	23.8	58.7
3	1.312	21.9	80.5
4	1.126	18.8	99.3
5	3.250E-02	0.5	99.9
6	8.938E-03	0.1	100.0

Vectors

Factor	1	2	3	4	5	6
TB	0.6593	-0.2043	0.0272	0.1429	0.2179	-0.6746
U3B	-0.6574	0.1992	-0.0376	-0.1655	0.1554	-0.6893
U4B	-0.1368	-0.6024	0.4388	-0.4030	0.4937	0.1406
U5B	-0.2475	-0.0605	0.1435	0.8608	-0.4056	0.0965
U6B	-0.0021	-0.1319	-0.8535	-0.1047	0.4734	0.1383
U7B	0.2305	0.7312	0.2373	-0.1947	0.5441	0.1480

2) The second principal component was composed of the variance-controlled utilization rates for nodes 4 and 7. The interpretation for this factor pattern was similar to the interpretation of the third and fourth principal components found in Case 1.

3) The third principal component had only one factor with a value greater than 0.5. This principal component is reflecting the contribution of the variance-controlled utilization rate of node 6 to the total variance.

4) The fourth principal component also only had one factor with a coefficient greater than 0.5. This principal component is reflecting the contribution of the variance-controlled utilization rate of node 5 to the total variance.

Factor scores were again obtained by multiplying the eigenvectors of the principal components by the original variables. The factor scores of the four principal components of the variance-controlled outputs are shown on Table 8.

A scatter plot of the first two principal components of the variance-controlled outputs is shown in Figure 7. The points have the alphabetic notation of the design point it represents.

Table 8. Principal Components of the Variance-Controlled Outputs

Design Point	PCB1	PCB2	PCB3	PCB4
1	-3.3449	0.9384	-0.1871	-0.6946
2	-1.3525	-1.1938	0.8400	-1.2693
3	-1.7860	0.3627	0.3640	1.7046
4	-1.2374	-1.4958	1.4654	0.7373
5	-0.8554	-0.0075	-2.0467	-0.6711
6	-0.3982	-1.5708	-0.8928	-1.3735
7	-0.2494	-0.4826	-1.7237	1.4777
8	0.2629	-1.8831	-0.3891	0.7716
9	-0.1486	2.3481	0.6299	-0.8199
10	0.7985	0.3961	1.7400	-1.3679
11	0.2083	1.4971	0.6936	1.1578
12	1.2246	-0.2186	1.8251	0.5919
13	1.0424	1.3402	-1.3713	-0.7304
14	2.3628	-0.4618	-0.3465	-1.0602
15	1.2687	1.1129	-0.7747	1.1487
16	2.0661	-0.8150	0.0371	0.3733
17	0.1379	0.1336	0.1368	0.0242

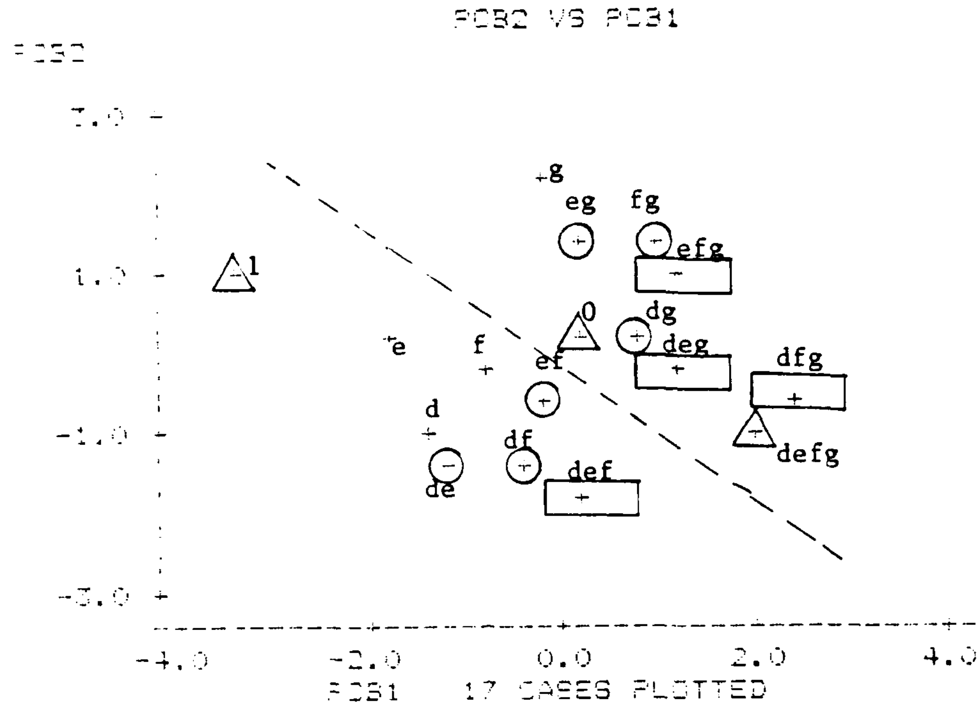

Figure 7. Scatter Plot of the First Two Principal Components of the Variance-Controlled outputs

Case 3. The result of the regression of the principal components and the control variables resulted in the following four equations:

$Z_1(\beta) = Z_1$

$Z_2(\beta) = Z_2 - (-1.63369*W1 + 3.78988*W7 - 3.00164*R6)$

$Z_3(\beta) = Z_3 - (-3.88701*W1 + 3.29738*W6)$

$Z_4(\beta) = Z_4$

where $Z_n(\beta)$ is the variance-controlled n-th principal component, Z_n is the original uncontrolled principal components, W_j is the working variable of node j, and R_j is

the routing variable of node j. (Appendix 5 contains the SAS outputs of the regression). Z_n was taken directly from the results of Case 1 and W_j and R_j were the means of the working and routing variables used in Case 2. Note that variance reductions were not realized for the first and the third principal components.

The variance-controlled principal components at each design point are shown in Table 9. PC(j)B stands for variance-controlled principal component j.

A scatter plot of the first two variance-controlled principal components is shown in Figure 8. The points have the alphabetic notation of the design point it represents.

Table 9. Variance-Controlled Principal Components

Design Point	PC1B	PC2B	PC3B	PC4B
1	3.4697	-0.7688	0.3163	-0.7841
2	1.2868	0.5086	-1.3008	-0.6216
3	1.8444	0.2873	0.3637	1.6190
4	0.9750	-0.0043	-0.6302	1.5305
5	0.3441	-0.6882	-0.6065	-1.4741
6	-0.7873	-0.3483	-1.0199	-1.3575
7	-0.5614	0.1110	0.9994	0.4702
8	-1.5218	-0.5868	-0.0964	0.4554
9	0.9338	-0.2272	0.8226	-0.6198
10	-0.3834	1.8325	0.0508	-0.5419
11	0.3085	0.3207	-0.1132	1.4648
12	-0.5000	0.6998	-0.5367	1.4634
13	-0.2788	-0.2093	1.4333	-1.3093
14	-1.1851	1.3792	-0.0791	-1.2938
15	-1.5326	-0.4577	-0.6285	0.4884
16	-2.4880	-0.3923	-0.0290	0.4313
17	0.0761	-0.1076	0.3146	0.0891

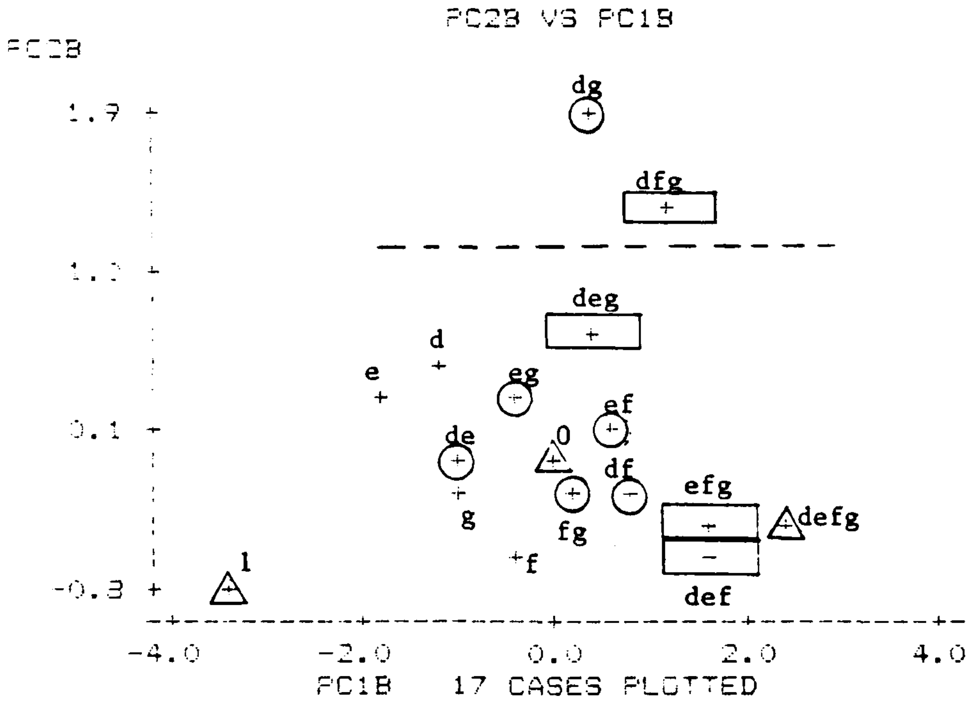

Figure 8. Scatter Plot of the First Two Variance-Controlled Principal Components

Observations

Both sets of principal components, one found using the original outputs and the other using the variance-controlled outputs, explained relatively equal percentages of the total variance of the original data--about 99%. Similarly, each individual principal component of the original outputs explained a relatively equal percentage of the total variance as its counterpart principal component of the variance-controlled outputs. Further, the last three principal components in each set had relatively equal eigenvalues or explained equal percentages of the total variance.

Several explanations can be given for these results. The delays that nodes 4 through 7 contributed to the customer's mean time in system were equal. This condition did not allow any node's effects to "overpower" the effects of the other nodes. This characteristic may have led to the equal percentages of the variance explained by the last three principal components.

Also, the model chosen may not have necessarily yielded orthogonal principal components. The fact that the utilization rate of node 7 loaded with the utilization rate of node 6 on the second principal component of the original outputs and then loaded with the utilization rate of node 4 on the second principal component of the variance-controlled outputs imply that the principal components were not orthogonal. That the utilization rate of node 7 "wavered" back and forth as to what other node utilization rate it loaded with suggest that principal components analysis of the outputs did not lead necessarily to orthogonal axes.

Given the earlier discussion about the individual principal components in each case, note that the plots that were shown for the principal components of the original outputs and the principal components using variance-controlled outputs did not have "parallel" axes. For both plots, the first principal component was a contrast of the time in system and the CPU utilization rate. However, the second principal components of the original outputs was loaded with the utilization rates of nodes 6 and 7 while the second principal component of the variance-controlled outputs was loaded with the variance-controlled utilization rates of nodes 4 and 7. Interestingly enough, the third principal component of the the original outputs had similar loadings as the third principal component of the variance-controlled outputs.

To facilitate a more meaningful discussion, the plot of the principal components of the original outputs was redrawn to contain the first and third

principal components. This plot should still be representative of the variance explained by the first and second principal components since, as observed previously, the last three principal components had relatively equal amount of variances explained. This plot is shown in Figure 9.

The following discussion results from a comparison of the plots of the principal components of the original outputs, Figure 9, the variance-controlled outputs, Figure 7, and the variance-controlled principal components, Figure 8:

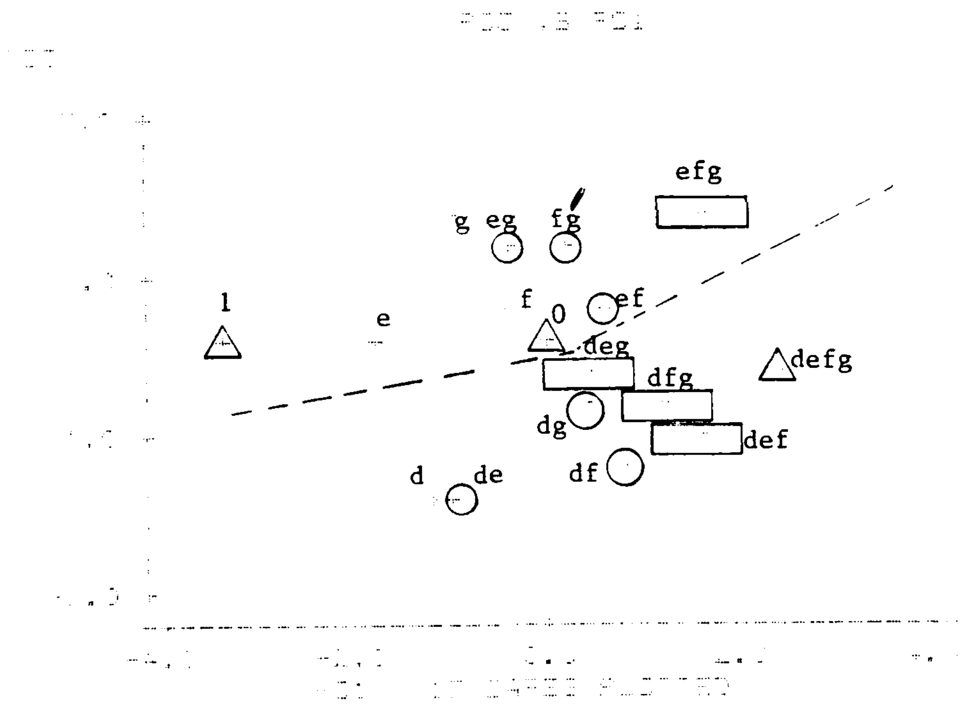

Figure 9. Scatter Plot of the First and Third Principal Components

1) The design point in which all the significant nodes were at their high, mean, and low levels, respectively, were labeled with triangles. The design points which

have just three nodes at their high levels, 20% above the mean, were labeled with rectangles. The design points which have just two nodes at their high levels, were labeled with circles.

2) The points representing all the significant nodes at their high and low levels occured on opposite sides of the cluster of points. The point representing all the nodes at their mean levels seemed to be in the middle of the other two points. This pattern presented itself more clearly with the principal components of the original outputs.

3) The points seemed to be progressing along the first principal components axis as the number of nodes at their high levels increase--the rectangles, as a whole, were ahead of the circles, as a whole, which were ahead of the unlabeled points (which represent having only one node at its high level.) This trend appeared more marked for the principal components of the variance-controlled outputs. The observation, mentioned previously, about the nonorthogonality of the axes seem to be supported by this observation of the plots. The progression of points as the number of nodes at its high level increase appeared to be along a path that is a linear combination of the axes, not merely along just the first principal component axis.

4) A dashed line drawn on the plot of the principal components of the original outputs separates those points with node 4 at its high level and node 4 at its low level. Those points with node 4 at its high level seemed to be farther up on the third principal component axis. The factor loadings on this principal component were divided between the utilization rates of nodes 4 and node 7. The factor loadings were opposite in sign and the loading for node 4 was higher in magnitude. Apparently node 4 at its high level tended to drive this principal component down.

5) A dashed line drawn on the plot of the principal components of the variance-controlled outputs separates those points with node 7 at its high level and node 7 at its low level. Those points with node 7 at its high level seemed, as a whole, to be farther up on the second principal component axis. The factor loadings on this principal component were divided between the variance-controlled utilization rates of nodes 4 and 7. The factor loadings were opposite in sign and the loading for node 7 was higher in magnitude. Apparently, node 7 at its high level drives this principal component up. This pattern of division along the level of a factor is more clearly seen with the plot of the principal components of the variance-controlled outputs.

6) In the plot of the variance-controlled principal components, the points "dg" (nodes 4 and 7 at their high levels) and "dfg" (nodes 4, 6, and 7) at their high levels seemed to be separated from the main cluster of points.

V. Factor Analysis

Since the principal components analysis presented unclear results, a factor analysis analogous to the principal components analysis was performed. Factor analysis is similar to principal components analysis in that as few as possible new variables are used to explain the variance of the original data. In the case of principal components analysis, the total variance of the original data is considered. In factor analysis, however, only the part of the total variance that is shared by the original variables are explored.

The interpretation of factors (the new variables) in factor analysis is often complicated by having many factors with moderate-sized loadings, all of which are significant. In other words, an original variable loads on more than one factor. A concept called factor rotation attempts to remedy the problem. A factor rotation minimizes the number of significant loadings on each row of the pattern matrix (op**mally, one significant loading per original variable) and to maximize the number of loadings with negligible values (Dillon,1984:69).

A type of rotation used in this research was a VARIMAX rotation. In this rotation, the variation of the squared factor loadings within a factor is maximized (Kaiser,1958:189-200).

The three cases of factor analysis data that were compared were 1) the factors of the outputs of a model on which no control variates had been applied, 2) the factors of the outputs of a model on which control variates were first applied and, 3) the variance-controlled factors.

Case 1 Methodology. Table 3 showed the original data matrix--T stood for time in system and U(j) stood for utilization rate at node j. Nodes 1 and 2 had zero utilization rates. Factor analysis was performed using this original data matrix.

Case 2 Methodology. For this case, variance reduction was performed before factor analysis of outputs was completed. The actual steps taken to perform the Case 2 factor analysis were equivalent to the steps taken to perform the Case 2 principal components analysis.

Case 3 Methodology. Case 3 followed directly from Case 1 and used the same methodology as Case 2. Since the factors were already found, all that was required for this case was to run the control variates regression using the factors found in Case 1 and the same input variable information used in Case 2. The steps used to find the variance-controlled factors were similar to finding the variance-controlled principal components.

Case 1 Results. The SAS software system was used to perform the factor analysis. The outputs of the initial factor analysis without any rotation of the axes were very similar to the Case 1 principal components analysis outputs.

The table of eigenvalues showed four factors with eigenvalues greater than one. The four factors explained 99.4% of the variance of the data.

The eigenvectors and factor pattern matrix were similar to the eigenvectors and pattern matrix of the principal components analysis.

After a VARIMAX rotation, the above factor patterns changed for the third and fourth factors. The rotated third factor had a significant loading of the utilization rate of node 4 only while the rotated fourth factor had a significant loading of the utilization rate of node 5 only. Table 10 shows the rotated factor pattern for Case 1.

Factor scores were obtained by multiplying the eigenvectors of the factors by the original variables. Factor scores are given by the following equation

$$\begin{aligned} Y_j &= a_{ij}X_1 + a_{2j}X_2 + \dots + a_{pj}X_p \\ &= a_j^T X \end{aligned}$$

where Y_i is the factor score, α_i^T is the eigenvector, and X is the vector of original variables.

Table 10. Rotated Factor Pattern for Case 1

	Factor1	Factor2	Factor3	Factor4
T	-0.9928	-0.0495	-0.0413	-0.0591
U3	0.9962	0.0010	0.0032	0.0386
U4	0.0331	-0.0586	0.9838	-0.1058
U5	0.0773	-0.0624	-0.1369	0.9833
U6	-0.0412	-0.7822	-0.4768	-0.3919
U7	0.0270	0.8964	-0.2983	-0.3177

The factor scores of the four factors were calculated for each design point. Table 11 shows the factor scores for Case 1. Scatter plots of the first factor against each of all the other factor are shown in figures 10 through 12. Each point on the plot has the alphabetic notation (shown in Table 2) to indicate which design point it represents.

Case 2 Results. The result of the regression to find the variance-controlled outputs were shown on Table 4. The values are labelled TB, U3B, U4B, U5B, U6B, and U7B for time in system and utilization rates of the node j, respectively, to indicate that control variates had been performed .

Again, factor analysis was performed, this time on these variance-controlled outputs. The eigenvalues and factor patterns obtained without the rotation were similar to the principal components analysis outputs of Case 2. The table showed four factors with eigenvalues greater than one. The four factors accounted for 99.3% of the variance.

The eigenvectors and loadings matrix also were similar to the eigenvectors and loadings matrix of the Case 2 principal components.

Table 11. Factor Scores of Case 1 Factors

Design Point	Fact1	Fact2	Fact3	Fact4
1	2.5468	-0.0162	0.0214	-0.1025
2	0.8546	-0.2267	1.7748	-0.4211
3	0.9991	-0.1395	-0.3007	1.8666
4	0.2392	-0.2235	1.5593	1.5100
5	0.7477	-1.2966	-0.8144	-0.9033
6	-0.2269	-1.2543	0.8362	-1.1327
7	-0.2005	-1.4923	-1.1807	0.7935
8	-1.0464	-1.2981	0.4606	0.4367
9	0.6622	1.7745	-0.8579	-0.7801
10	-0.4142	1.5761	0.8638	-1.0224
11	-0.1183	1.2160	-0.8578	1.1574
12	-0.8443	1.2318	0.8091	0.8731
13	0.1920	0.0731	-1.3334	-1.1645
14	-0.5958	-0.1680	0.5059	-1.3903
15	-1.0002	0.0479	-1.5649	0.2837
16	-1.8037	-0.0265	0.0555	-0.0417
17	0.0088	0.2233	0.0234	0.0375

After a VARIMAX rotation, the above patterns stayed the same except for the third factor. The rotated third factor was loaded with the variance-controlled utilization rates of node 6 and node 7. Table 12 shows the rotated factor pattern of the Case 2 factor analysis.

Factor scores were again obtained by multiplying the eigenvectors of the factors by the original variables. The factor scores of the four factors of the variance-controlled outputs are shown in Table 13.

Scatter plots of the first factor against each of all the other factors are shown in figures 13 through 15. The points have the alphabetic notation of the design point it represents.

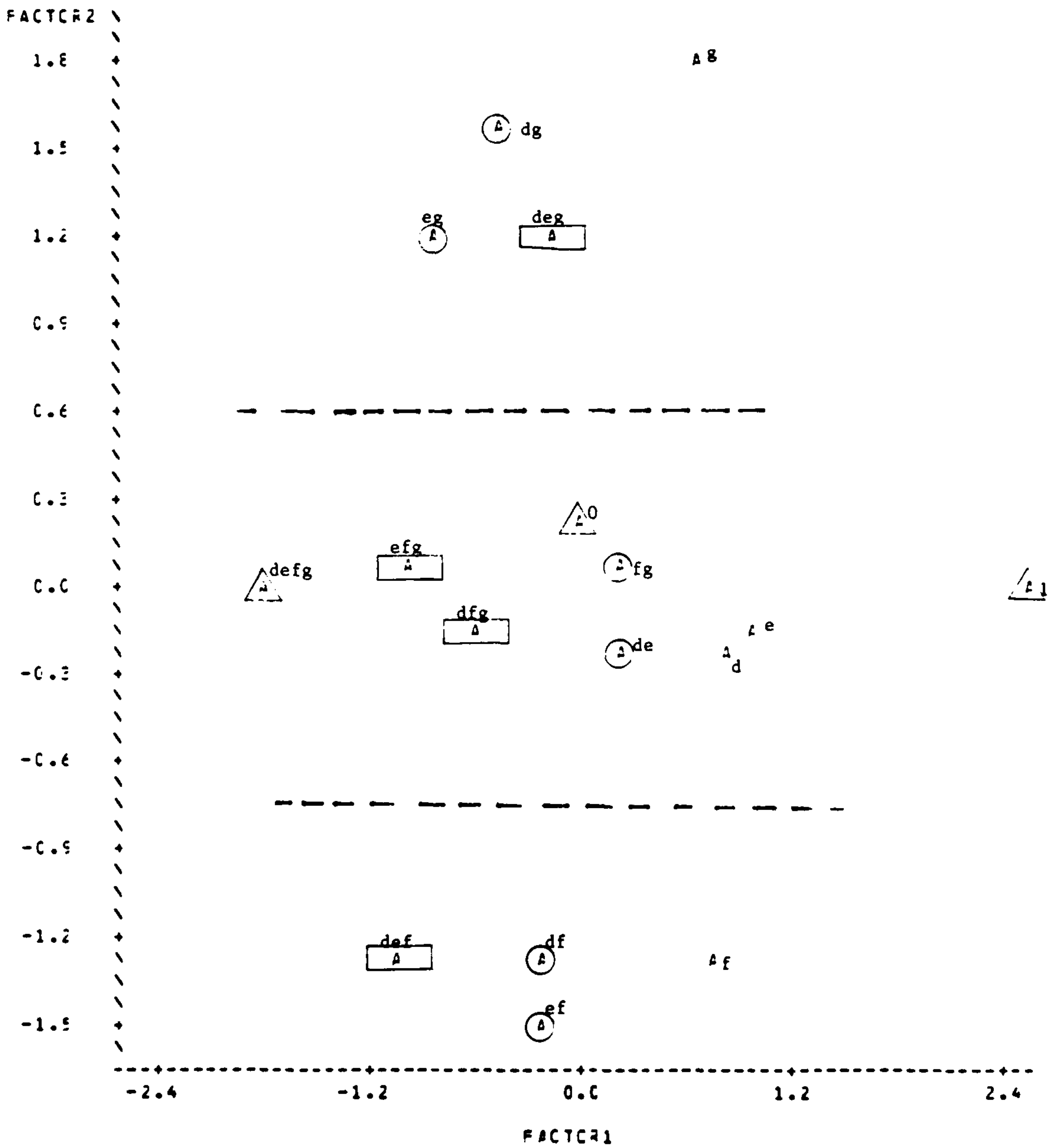

Figure 10. Scatter Plot of the First and Second Factors of the Original Outputs

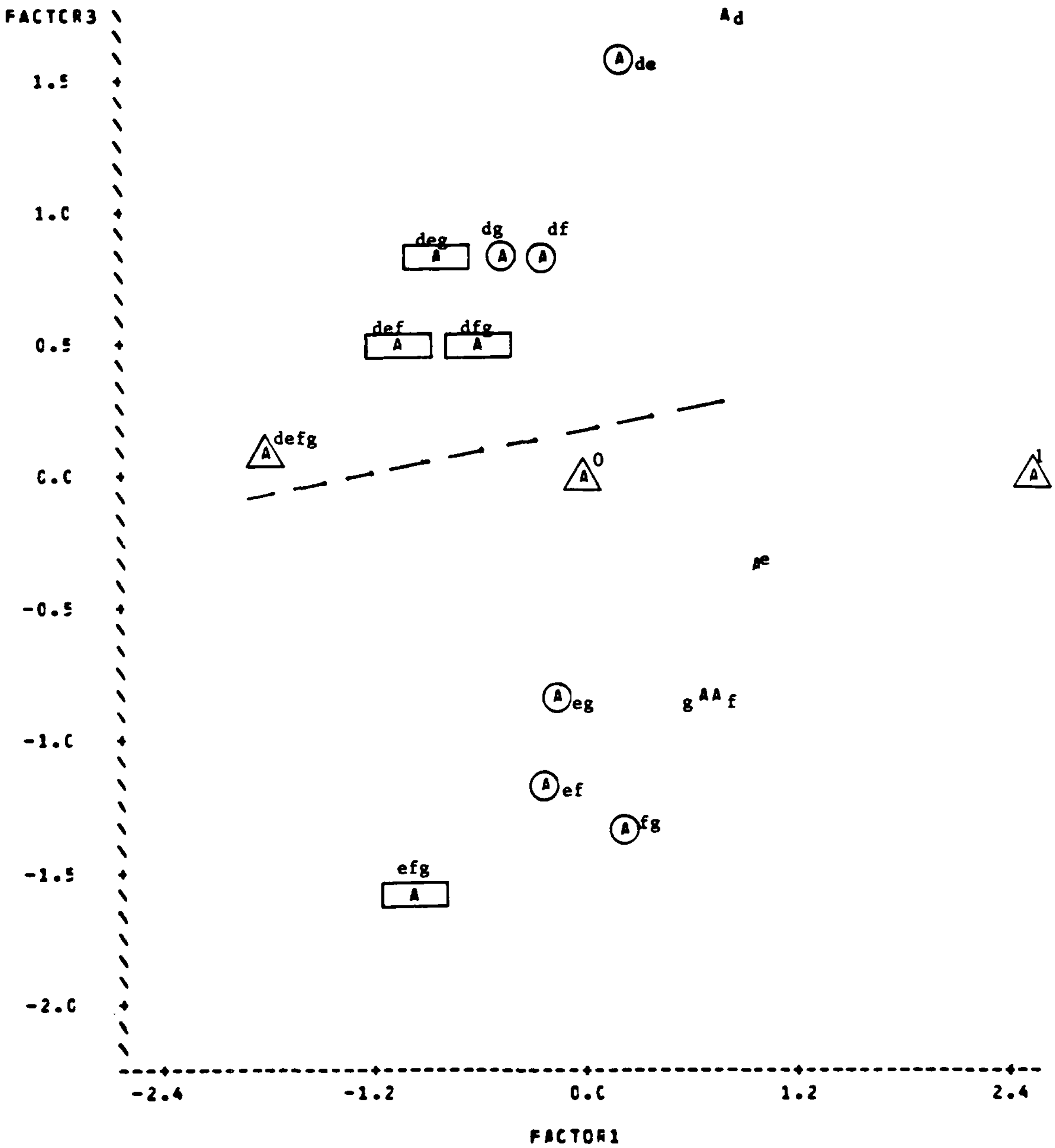

Figure 11. Scatter Plot of the First and Third Factors of the Original Outputs

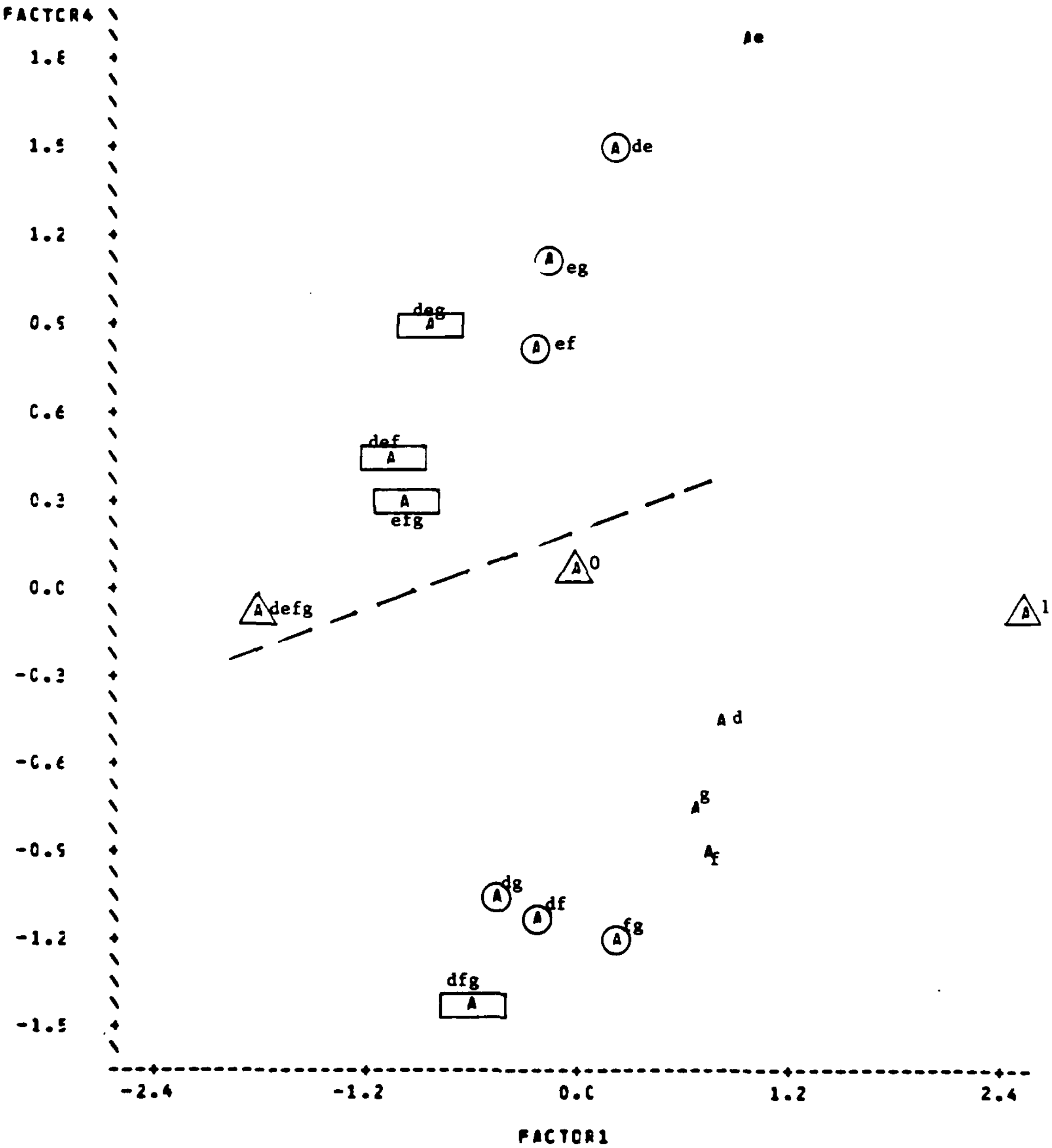

Figure 12. Scatter Plot of the First and Fourth Factors of the Original Outputs

Table 12. Rotated Factor Pattern for Case 2

	Factor1	Factor2	Factor3	Factor4
TB	-0.9936	-0.0286	-0.0048	-0.0791
U3B	0.9951	0.0363	0.0178	0.0560
U4B	0.0510	0.9462	-0.2510	-0.1760
U5B	0.1057	-0.0493	-0.1168	0.9835
U6B	0.0165	-0.1188	0.9779	-0.1477
U7B	-0.0258	-0.7173	-0.5392	-0.4924

Table 13. Factor Scores of Case 2 Factors

Design Point	Fact1	Fact2	Fact3	Fact4
1	2.5285	-0.0541	0.0364	-0.1212
2	0.8156	1.6336	-0.3483	-0.6217
3	0.8705	-0.3260	-0.3320	1.8228
4	0.2311	1.4995	-0.7967	1.2181
5	0.7609	-0.2197	1.7336	-0.5571
6	0.1946	1.3471	1.1539	-0.9519
7	-0.1997	-0.4144	1.5630	1.3205
8	-0.7680	1.0698	0.7899	0.8980
9	0.8077	-1.3602	-1.1117	-1.0195
10	-0.1977	0.3703	-1.5740	-1.3157
11	-0.0384	-1.2598	-0.9628	1.7967
12	-1.0285	0.2937	-1.5052	0.4361
13	-0.1479	-1.3307	0.7646	-1.1440
14	-1.3995	0.1534	0.3219	-1.3484
15	-0.7585	-1.5133	0.3174	0.5480
16	-1.6041	0.2027	0.1022	0.0570
17	-0.0665	-0.0918	-0.1524	-0.0173

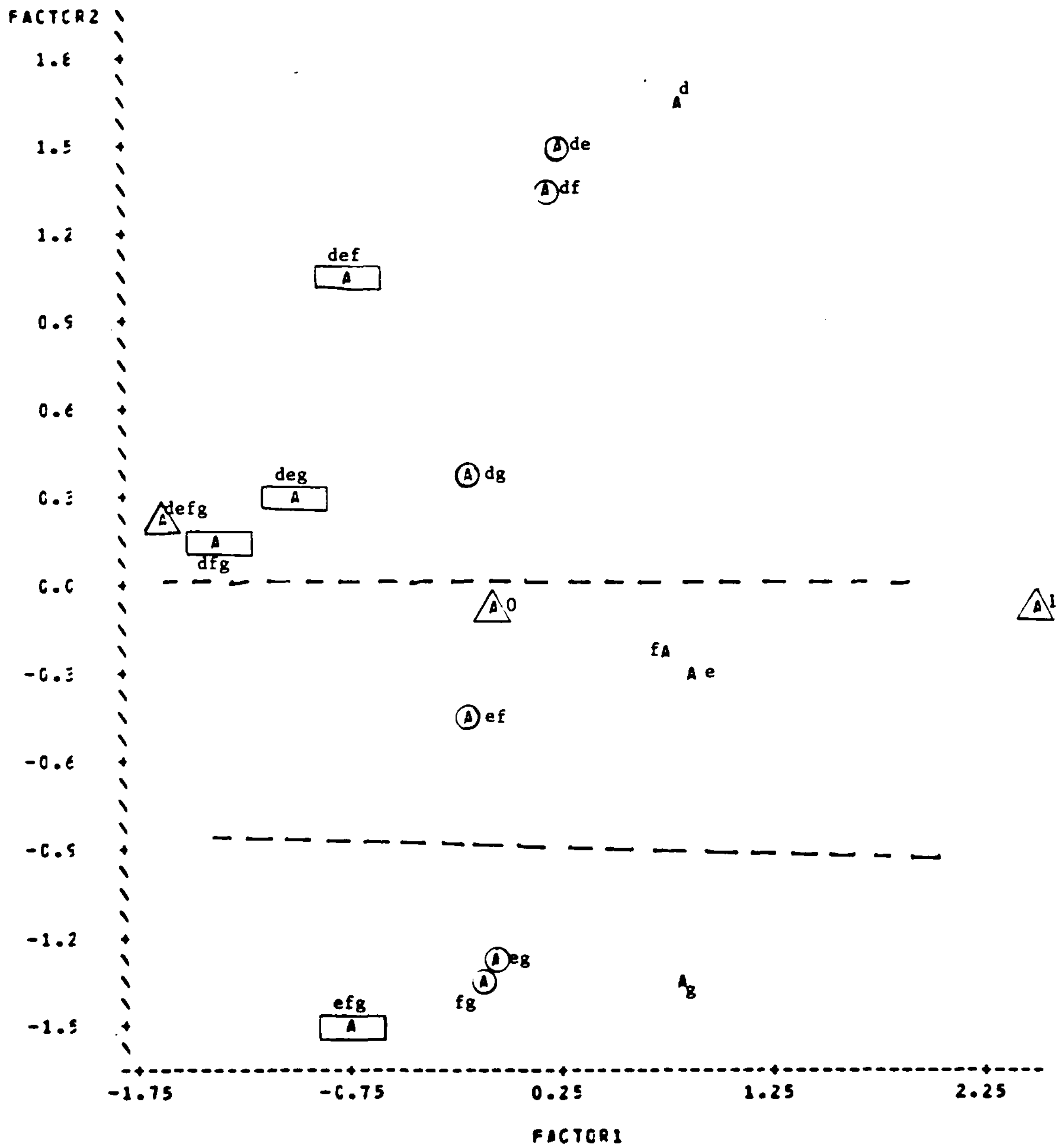

Figure 13. Scatter Plot of the First and Second Factors of the Variance-Controlled Outputs

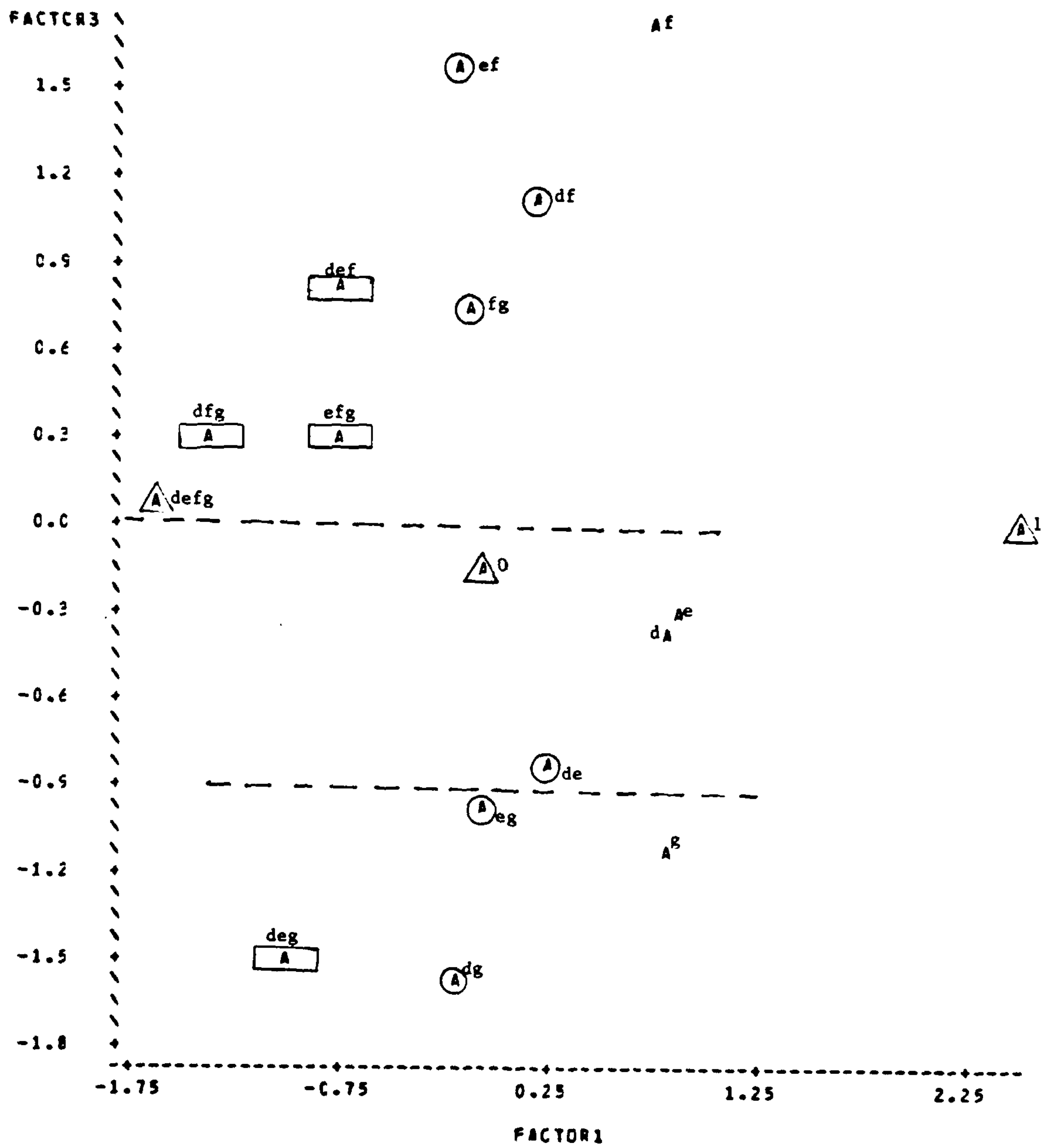

Figure 14. Scatter Plot of the First and Third Factors of the Variance-Controlled Outputs

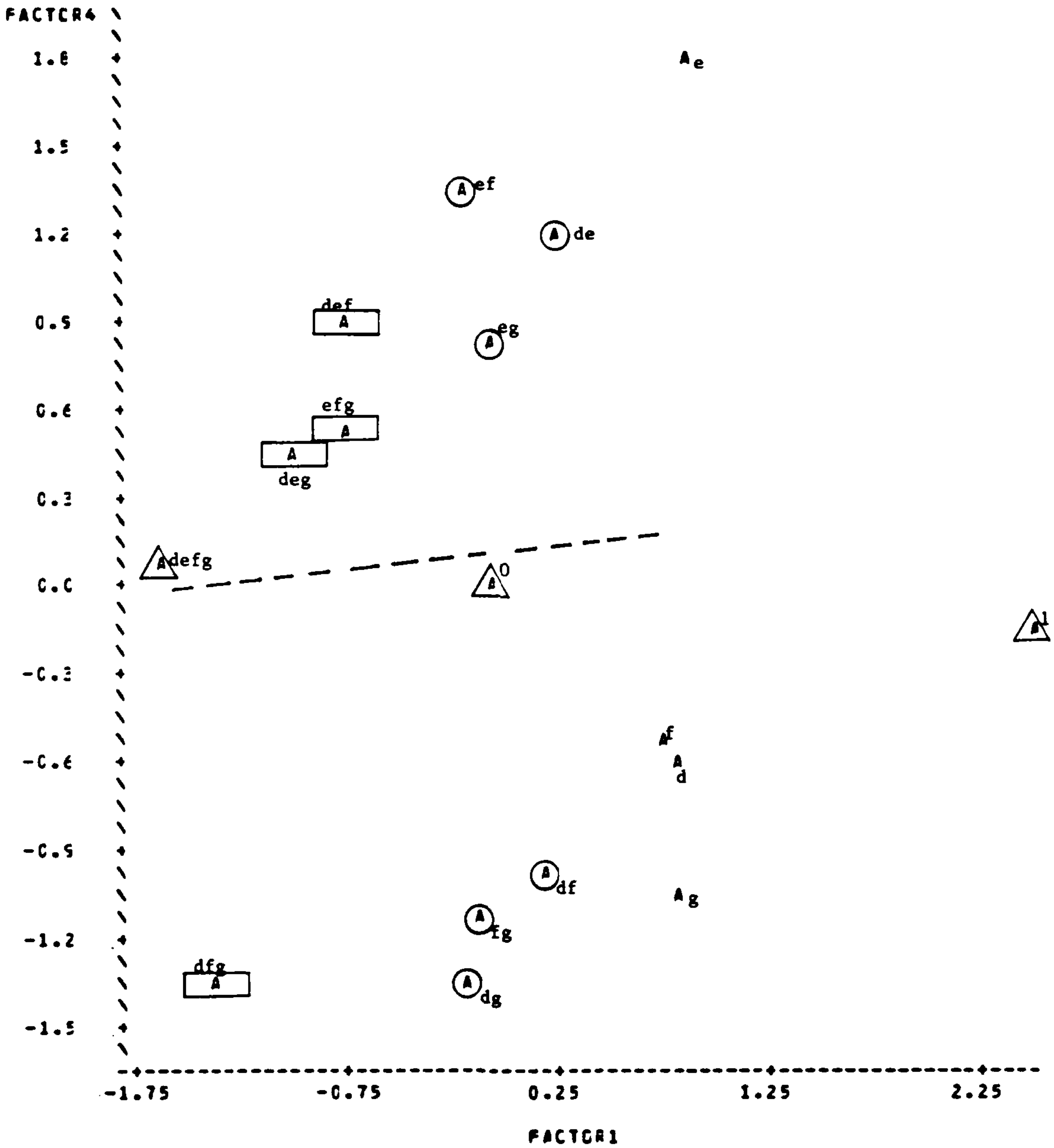

Figure 15. Scatter Plot of the First and Fourth Factors of the Variance-Controlled Outputs

<u>Case 3 results</u>. The result of the regression of the factors and the means of the control variables resulted in the following four equations:

$$Z_1(\beta) = Z_1 - (2.06037*W_5)$$

$$Z_2(\beta) = Z_2 - (-2.77119*W_1 + 2.09686*W_7 + 2.21997*R_1)$$

$$Z_3(\beta) = Z_3 - (2.58496*W_1 - 1.57872*W_5 - 3.39355*W_6)$$

$$Z_4(\beta) = Z_4$$

where $Z_n(\beta)$ is the variance-controlled n-th factor, Z_n is the original non-controlled factor, W_j is the working variable of node j, and R_j is the routing variable of node j. Note that variance-reduction was not realized for the fourth factor.

The variance-controlled factor scores at each design points are shown in Table 14. The scatter plot of the first variance-controlled factor against each of all the other variance-controlled factors are shown in figures 16 through 18.

<u>Observations</u>

Both sets of factors, one found using the original outputs and the other using the variance-controlled outputs, explained relatively equal amount of variances of the original data--about 99%. Similarly, each individual factor of the original outputs explained a relatively equal percentage of the variance as its counterpart factor of the variance-controlled outputs. Further, the last three factors in each set had relatively equal eigenvalues or explained equal amounts of the variance.

The following discussion result from a comparison of the plots of the factors of the original outputs, the plots of the factors of the variance-controlled outputs, and the plots of the variance-controlled factors.

Table 14. Variance-Controlled Factor Scores

Design Point	Fact1	Fact2	Fact3	Fact4
1	2.5736	0.0762	0.0129	-0.1025
2	0.5657	-0.3853	1.3182	-0.4211
3	0.8658	0.7934	-0.6022	1.8666
4	0.4071	-0.2466	0.3347	1.5100
5	0.5767	-0.9516	0.0079	-0.9033
6	-0.5110	-0.9493	0.8061	-1.1327
7	-0.8190	0.1399	-0.7481	0.7936
8	-0.0607	-0.2678	-0.7675	0.4367
9	0.5338	0.1120	-0.4682	-0.7801
10	-0.0339	0.9714	0.3836	-1.0224
11	-0.4397	0.2478	0.3626	1.1574
12	-0.2833	0.7116	0.4699	0.8731
13	0.2613	-0.2417	-1.0108	-1.1646
14	-0.4831	0.6094	-0.3896	-1.3903
15	-0.6310	-0.2336	0.0255	0.2837
16	-1.7575	-0.2617	-0.0506	-0.0417
17	0.0546	-0.6538	-0.5398	0.0375

1) For all the plots, the design points for the significant nodes at their high, mean, and low levels were labeled with triangles. The design points which have three nodes at their high levels, 20% above the mean, were labeled with rectangles. The design points which have two nodes at their high levels were labeled with circles.

2) The points representing all the significant nodes at their high and low levels occur on opposite sides of the cluster of points. The point representing the nodes at their mean levels seem to be in the middle of the other two points.

3) The points seem to be "regressing" along the first factor axis as the number of nodes at their high levels increase--the rectangles, as a whole, are behind the

circles, as a whole, which are behind the unlabeled points (which represent

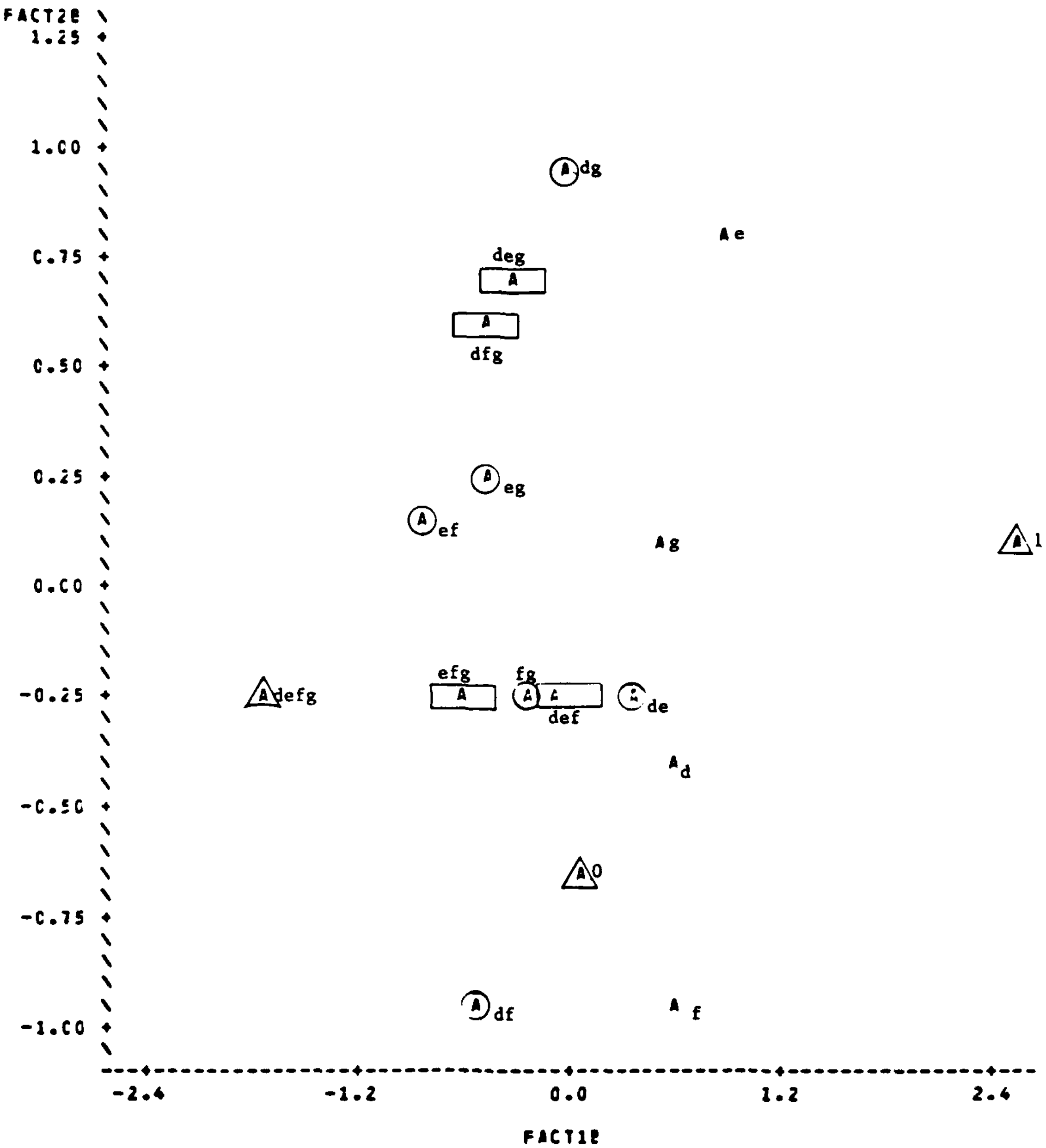

Figure 16. Scatter Plot of the First and Second Variance-Controlled Factors

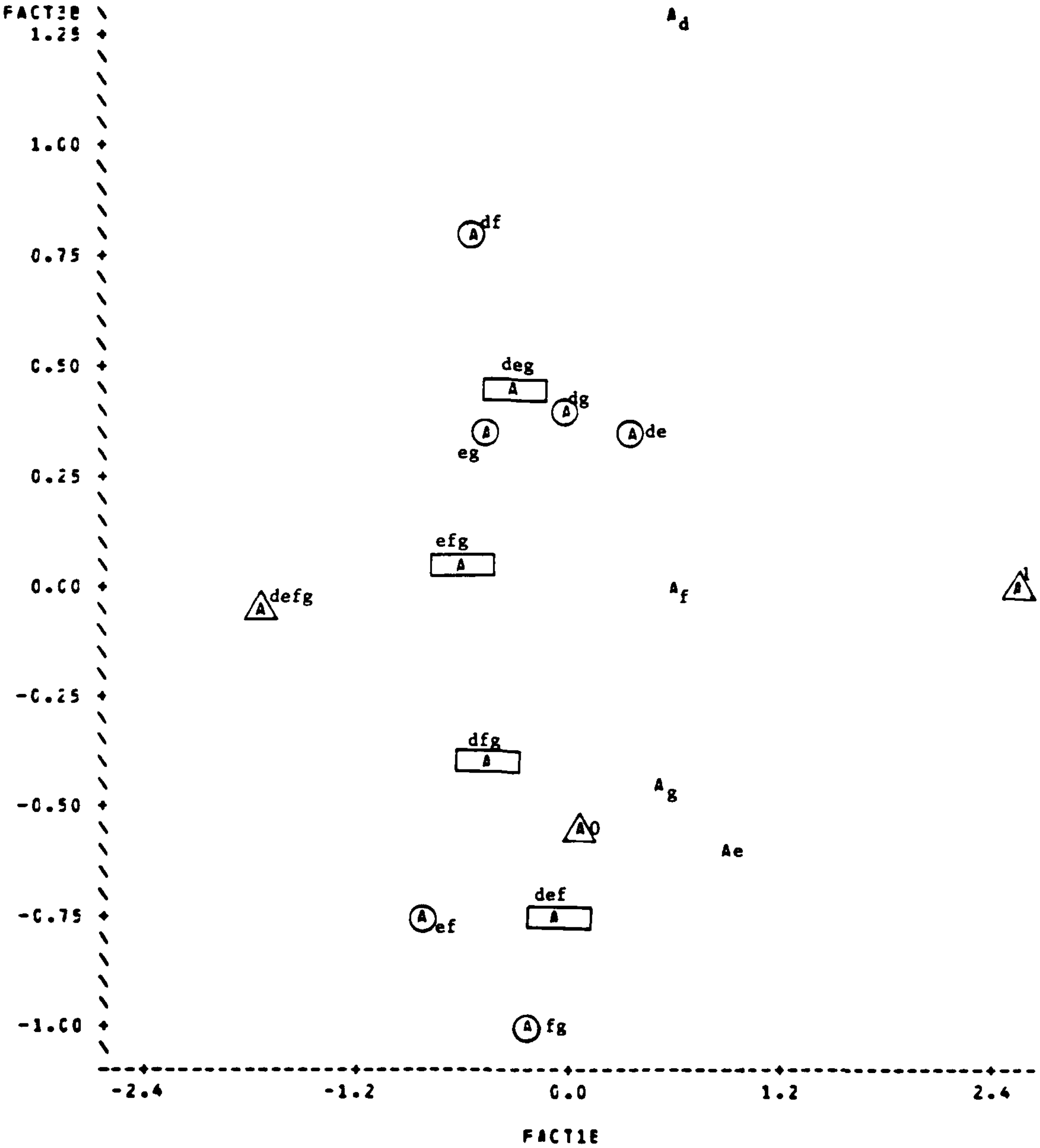

Figure 17. Scatter Plot of the First and Third Variance-Controlled Factors

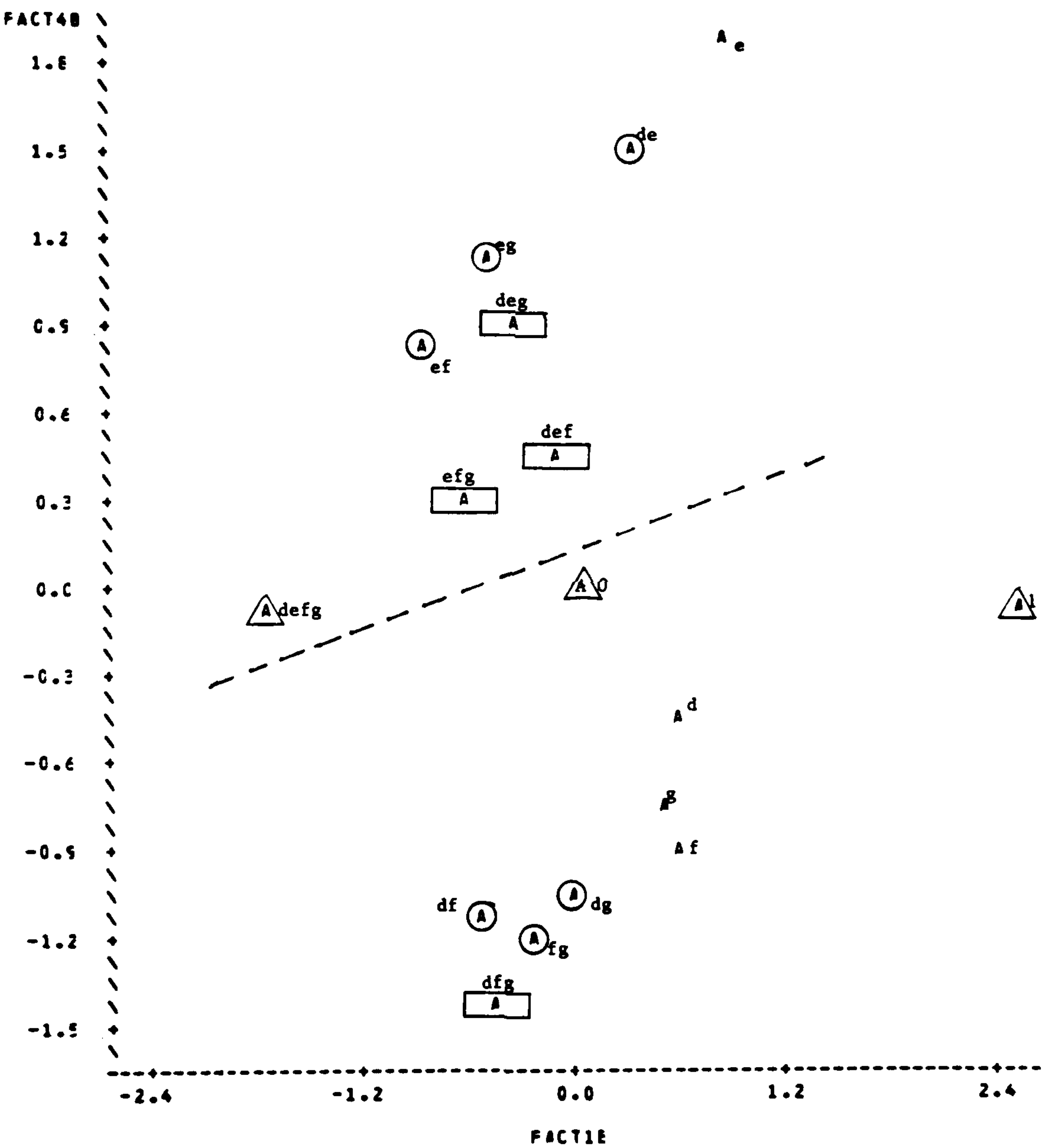

Figure 18. Scatter Plot of the First and Fourth Variance-Controlled Factors

having only one node at its high level.) This trend appears very clearly for the factor 1 of the variance-controlled outputs.

4) The following observations are made about the Case 1 plots:

a) The plot of factor 1 against factor 2 seemed to have three groupings of points--those points with node 7 at its high level (characterized by the presence of the letter "g"), those points with node 6 at its high level (letter "f") and all the other points. The group of points with node 7 at its high level was the farthest up the factor 2 axis, while the group of points with node 5 at its high level was the farthest down the factor 2 axis. The group of all the other points was between the other two groups. This pattern in the plot was reflective of the factor pattern of factor 2. The rotated factor pattern of factor 2 had significant loadings for the utilization rates of node 6 and 7. The loadings are opposite in sign and the loading for node 7 was bigger in magnitude. Apparently, node 7 at its high level resulted in high factor scores for factor 2 while node 6 at its high level resulted in low factor scores for factor 2.

b) The plot of factor 1 against factor 3 seemed to have two groupings of points--those points with node 4 at its high level (presence of the letter "d") and those points with node 4 at its low level (absence of the letter "d"). The group of points with node 4 at its high level was above the group of points with node 4 at its low level. This pattern in the plot was reflective of the factor pattern of factor 3. The rotated factor pattern of factor 3 had a significant loading only for the utilization rate of node 4. Apparently, node 4 at its high level resulted in higher factor 3 scores while node 4 at its low level resulted in lower factor 3 scores.

c) The plot of factor 1 against factor 4 seemed to have two groupings of points--those point with node 5 at its high level (presence of the letter "e") and node 5 at its low level (absence of the letter "e"). The group of points with node 5 at its high level was above the group of points with node 5 at its low level. This

pattern in the plot was reflective of the factor pattern of factor 4. The rotated factor pattern of factor 4 had a significant loading only for the utilization rate of node 5. Apparently, node 5 at its high level resulted in higher factor 4 scores while node 5 at its low level resulted in lower factor 4 scores.

5) The following observations are made about the Case 2 plots:

a) The plot of the first and second factors of the variance reduced outputs seemed to have three groupings of points--those points with node 4 at its high level, those points with node 7 at its high level, and all the other points. The group of points with node 4 at its high level was the farthest up the factor 2 axis while the group of points with node 7 at its high level was the farthest down the factor 2 axis. All the other points were between the other two groups. This pattern in the plot was reflective of the factor pattern of factor 3. The rotated factor pattern of factor 2 was loaded with the variance-controlled utilization rates of node 4 and node 7. The loadings were opposite in sign and the loading for node 4 was bigger in magnitude. Apparently, node 4 at its high level resulted in high factor 2 scores while node 7 at its high level resulted in low factor 2 scores.

b) The plot of the first and third factors of the variance-controlled outputs seemed to have three groupings--those points with node 6 at its high level, those points with node 7 at its high level, and all the other points. The group of points with node 6 at its high level was farthest up the factor 3 axis while the group of points with node 7 at its high level was farthest down the factor 3 axis. All the other points were between the other two groups. This pattern in the plot was reflective of the factor pattern of factor 3. The rotated factor pattern of factor 3 was loaded with the variance-controlled rates of nodes 6 and 7. The loadings were opposite in sign and the loading for node 6 was bigger in magnitude. Apparently, node 6 at its high level resulted in high factor 3 scores while node 7 at its high level resulted in low factor 3 scores.

c) The plot of the first and fourth factors of the variance-controlled outputs seemed to have two groupings of points--those points with node 5 at its high level and those points with node 5 at its low level. This pattern in the plot was reflective of the factor pattern of factor 4. The rotated factor pattern of factor 4 was loaded solely with the variance-controlled utilization rate of node 5. Apparently, node 5 at its high level resulted in higher factor 4 scores while node 5 at its low level resulted in lower factor 4 scores.

6) The following observations are made about the Case 3 plots:

a) No discernible pattern was found in the plot of the variance-controlled factors 1 and 2.

b) No discernible pattern was found in the plot of the variance-controlled factors 1 and 3.

c) The plot of the variance-controlled factors 1 and 4 seemed to have two groupings of points--those points with node 5 at its high level and node 5 at its low level. The group of points with node 5 at its high level are above the group of points with node 5 at its low level. The interpretation of this plot pattern was similar to the interpretation of the plot of the first and fourth factor of the original variables.

7) Of all the plots, the Case 2 plots had the clearest distinction between the group of points with three nodes at their high levels, the group of points with two nodes at their high levels, and the group of points with just one node at its high level. There was a very clear separation along the factor 1 axis. An explanation for this is that the variance in the original data has been reduced. The "noise" that was in the original data was causing the points to stray to the other groups of points; control variates prevented this straying.

VI. Conclusions and Recommendations

The purpose of this thesis was to compare the variance-controlled and uncontrolled principal components of a specific model. The conclusions of this research are important because they add to the body of knowledge that an analyst draws upon when faced with problems of data and variance reduction.

This research was accomplished because the study of principal components analysis with the technique of control variates has not been performed before. It was also the hope of the researcher that, given actual systems, the results obtained from the study would make possible an assessment of real systems. For example, given two computer systems with different node mean service times and probabilities of transitions or two similar systems with different settings, an index or a couple of indices (the principal components) can be used to rate each system. A judgement such as degree of efficiency or need for modifications can then be implied from this index or set of indices. Control variates can then be performed to "fine tune" the index or set of indices.

The methods of comparison used in this study was to examine the percentage of the total variance explained by the principal components and to review the scatter plots of the first two principal components.

In Case 1, the four principal components were found to be significant and explained 99.4% of the total variance. In Case 2, four principal components were also found to be significant and explained 99.3% of the total variance. For each set of principal components, the last three had relatively equal eigenvalues--the proportion of variance each explained were equal. The scatter plots yielded by the three cases of data presented similar patterns.

One can conclude that the results obtained from the principal components analysis are inconclusive and ambiguous. However, several explanations can

be volunteered for the results. The network model had several unfortunate characteristics that resulted in the unclear outcomes. The nodes 4 through 7 contributed equal delays to a customer's mean time in system. None of these nodes' effects were allowed to override any of the other node effects. The equal-delay characteristic may also have resulted in the equal-valued principal components. Also, the model routings and configurations seemed to have created nonorthogonal principal components. Node 7 loaded with either node 4 or 6 depending on the conditions. The prinicpal components plots had characteristics that support this explanation.

A factor analysis of the same data generated some different results. The factor patterns were similar for both types of analyses but the plots were different. Factor analysis of the variance-controlled outputs yielded more distinct "breaks" between the groups of points along the first factor axis; control variates was at work. Also, the plots of the factors of the variance-controlled outputs are more easily interpreted than the plots of the factors of the variance-controlled factors. Apparently, control variates perfromed before factor analysis resulted in less noise in the final data--results, in terms of plots, are better if variance reduction is accomplished before factor analysis. More importantly, the factor patterns of the factors were more easily observed on the plots than the factor patterns of the principal components.

The observations and conclusions that were made suggest the following recommendations.

Recommendations

As was mentioned, the "metrics" used in the comparisons were percentage of the total variance explained by the principal components and a review of the scatter plots. If other modes of measurements or measures of effectiveness (MOEs) were used in the comparison, more meaningful results may have resulted. Explanation for the differences, in addition to finding the differences, could have been made.

Another recommendation is to change the technique used. Case 1 results found four principal components. The last three principal components explained relatively equal percentages of the total variance. There may have been other types of analyses that could have produced clearer results.

Another recommendation is to select another model for study. This change may be either changes in the parameters values such as mean service times or probabilities of transition or a different model entirely. The results the current model is producing are unclear. The system makes it difficult to determine which node or nodes are critical to forming the principal components or to reducing variance.

A final recommendation is to select a different set of outputs to study. Perhaps the outputs used in this research are not as conducive to the types of analysis used as another set might have been.

Appendix 1. SLAM Code for the Network Model

```
c    program main(input,output,tape5=input,tape6=output,tape7,tape1,
c    1tape2,tape3,tape4)
c ********************************************
c *  Main Program - Begin                     *
c ********************************************
c
     program main
     dimension nset(5000)
     common qset(5000)
     common/scom1/ atrib(100),dd(100),ddl(100),dtnow,ii,mfa,mstop,nclnr
    1,ncrdr,nprnt,nnrun,nnset,ntape,ss(100),ssl(100),tnext,tnow,xx(100)
     common/ucom1/ depart(10),rmean(10),p(10,10),servt(10),ecount(2)
     common/ucom2/ isubcap,nusssn,numcust,tclear,nstudy
     common/ucom7/ numexp, delta, xmean(10), z
     equivalence (nset(1),qset(1))
     integer numexp, z
     real delta, xmean, rmean
     nnset=5000
     ncrdr=5
     nprnt=6
     ntape=7
     z=0
c
     read (ncrdr,*) isubcap,nusssn,numcust,tclear,nstudy
c
     do 10 i=1,nusssn+2
       read (ncrdr,*) (p(i,j),j=1,nusssn+2)
10   continue
c
     open(unit=10,file='p750.edm',status='old')
     read(10,*) numexp, delta, (xmean(i),i=1,nusssn+2)
c
     open(unit=15,file='p750.op',status='new')
c
     call slam
     close(10)
     close(15)
     stop
     end
c
c ********************************************
c *  Main Program - End                       *
c ********************************************
c
c ********************************************
c *  SUBROUTINES                              *
```

```
c *********************************************
c
c *********************************************
c *  Subroutine EVENT                         *
c *********************************************
c
      subroutine event(i)
      common/scom1/ atrib(100),dd(100),ddl(100),dtnow,ii,mfa,mstop,nclnr
     1,ncrdr,nprnt,nnrun,nnset,ntape,ss(100),ssl(100),tnext,tnow,xx(100)
      common/ucom1/ depart(10),rmean(10),p(10,10),servt(10),ecount(2)
      common/ucom2/ isubcap,nusssn,numcust,tclear,nstudy
c
      ecount(1)=ecount(1)+1
      if(tnow.gt.tclear) ecount(2)=ecount(2)+1
c
      goto (1,2),i
c
1     call arss
      return
2     call endss
      return
      end
c
c *********************************************
c *  Subroutine INTLC                         *
c *********************************************
c
      subroutine intlc
      common/scom1/ atrib(100),dd(100),ddl(100),dtnow,ii,mfa,mstop,nclnr
     1,ncrdr,nprnt,nnrun,nnset,ntape,ss(100),ssl(100),tnext,tnow,xx(100)
      common/ucom1/ depart(10),rmean(10),p(10,10),servt(10),ecount(2)
      common/ucom2/ isubcap,nusssn,numcust,tclear,nstudy
      common/gcom5/ iseed(10),jjbeg,jjclr,mmnit,mmon,nname(5),nncfi,
     &nnday,nnpt,nnprj(5),nnrns,nnstr,nnyr,sseed(10),lseed(10)
      common/ucom3/ multino(7)
      common/ucom7/ numexp, delta, xmean(10), z
      common/ucom9/ level(10)
      integer lseed(2000), numexp, z
      real delta, xmean, rmean, level
c
c *************************************
c * Read in parameter levels from     *
c * design matrix, and set parameters *
c * accordingly                       *
c *************************************
c
      read(10,*) (level(i),i=1,nusssn+2)
      do 10 i=1,nusssn+2
        rmean(i) = xmean(i) + (level(i)*(delta*xmean(i)))
```

```
10   continue
c
c **************************************
c * Create new seed for each run of     *
c * the experimental design matrix      *
c **************************************
c
     if ((((z*numexp)+1)-nnrun).eq.0) then
       j=(1.0e+12)*drand(1)
       do 15 i=nnrun, ((nnrun+numexp)-1)
         iseed(i)=j
15     continue
       z=z+1
     endif
     ised(2)=iseed(nnrun)
     x=drand(-2)
c
c **************************************
c * Initialize vectors and variables    *
c **************************************
c
     do 20 i=1,7
       multino(i)=0
20   continue
c
     do 25 i=1,2
       ecount(i)=0.
25   continue
c
     do 30 i=1,nusssn+3
       depart(i)=0.
30   continue
c
     do 35 i=1,nusssn+2
       servt(i)=0.
35   continue
c
c **************************************
c * Schedule each customer for          *
c * initial event                       *
c **************************************
c
     do 40 i=1,numcust
       etime=expon(rmean(1),2)
       atrib(1)=etime
       atrib(3)=i
       atrib(4)=1
       atrib(5)=2
       call schdl(1,etime,atrib)
```

```
40  continue
c
    do 45 i=1,nusssn+2
      xx(i)=0.
45  continue
    write(6,200)nnrun,numexp
200 format(1x,'SIMULATION STUDY IN PROGRESS : RUN ',i4, ' OF
   &'i4,' RUNS')
    return
    end
c
c ********************************************
c * Subroutine ENDSS                         *
c ********************************************
c
    subroutine endss
    common/scom1/ atrib(100),dd(100),ddl(100),dtnow,ii,mfa,mstop,nclnr
   1,ncrdr,nprnt,nnrun,nnset,ntape,ss(100),ssl(100),tnext,tnow,xx(100)
    common/ucom1/ depart(10),rmean(10),p(10,10),servt(10),ecount(2)
    common/ucom2/ isubcap,nusssn,numcust,tclear,nstudy
    common/ucom3/ multino(7)
c
    call schdl(1,0.,atrib)
    myq=atrib(4)
c
    if (nnq(myq).ne.0) then
      call rmove(1,myq,atrib)
      wait=tnow-atrib(2)
      call colct(wait,myq)
      rm=rmean(myq)
      service=expon(rm,2)
      atrib(4)=atrib(5)
      iat=atrib(4)+.00001
      call nextguy(iat,inext)
c
c ************************************
c * COLLECT STATISTICS WHILE PARKED  *
c * AT CPU                           *
c ************************************
c
      if (iat.eq.3) then
        multino(inext)=multino(inext)+1
      endif
c
      atrib(5)=inext
      call schdl(2,service,atrib)
      if (tnow.gt.tclear) then
        servt(myq)=servt(myq)+service
        depart(myq)=depart(myq)+1
```

```
      depart(nusssn+3)=depart(nusssn+3)+1
     endif
    else
     xx(myq)=0.
    endif
c
    if(myq.eq.3.and.nnq(2).gt.0.and.isubcap.ne.0.and.inext.eq.1
   &.and.nnq(myq).ne.0) then
     call rmove(1,2,atrib)
     service=0.
     atrib(4)=atrib(5)
     atrib(5)=3
     call schdl(1,service,atrib)
    endif
c
    return
    end
c
c *********************************************
c * Subroutine ARSS                             *
c *********************************************
c
    subroutine arss
    common/scom1/ atrib(100),dd(100),ddl(100),dtnow,ii,mfa,mstop,nclnr
   1,ncrdr,nprnt,nnrun,nnset,ntape,ss(100),ssl(100),tnext,tnow,xx(100)
    common/ucom1/ depart(10),rmean(10),p(10,10),servt(10),ecount(2)
    common/ucom2/ isubcap,nusssn,numcust,tclear,nstudy
    common/ucom3/ multino(7)
c
    iat=atrib(5)
c
    if (iat.eq.1) then
     resp=tnow-atrib(1)
     call colct(resp,1)
     rm=rmean(1)
     service=expon(rm,2)
     atrib(1)=tnow+service
     atrib(4)=1
     atrib(5)=2
     call schdl(1,service,atrib)
     if (tnow.gt.tclear) servt(iat)=servt(iat)+service
     go to 101
    endif
c
    if (iat.eq.2) then
     if (isubcap.ne.0) then
      numsub=0
      do 10 i=3,nusssn+2
       numsub=numsub+nnq(i)+xx(i)
```

```
10      continue
       if (numsub.lt.isubcap) then
        if (nnq(2).eq.0) then
         wait=0.
         call colct(wait,2)
         service=0.
         atrib(4)=2
         atrib(5)=3
         call schdl(1,service,atrib)
         go to 101
        else
         atrib(2)=tnow
         call filem(2,atrib)
         call rmove(1,2,atrib)
         wait=tnow-atrib(2)
         call colct(wait,2)
         atrib(4)=2
         atrib(5)=3
         service=0.
         call schdl(1,service,atrib)
         go to 101
        endif
       else
        atrib(2)=tnow
        call filem(2,atrib)
        return
       endif
      endif
     endif

100   if (xx(iat).gt.0.) then
       atrib(2)=tnow
       call filem(iat,atrib)
       return
      else
       wait=0.
       call colct(wait,iat)
       rm=rmean(iat)
       atrib(4)=iat
       call nextguy(iat,inext)
c
c ****************************************
c * COLLECT STATISTICS WHILE PARKED       *
c * AT CPU                                *
c ****************************************
c
      if (iat.eq.3) then
       multno(inext)=multno(inext)+1
      endif
```

```
c
      atrib(5)=inext
      service=expon(rm,2)
      xx(iat)=1
      call schdl(2,service,atrib)
      if(tnow.gt.tclear) servt(iat)=servt(iat)+service
     endif
c
101  if (tnow.gt.tclear) then
      depart(iat)=depart(iat)+1
      depart(nusssn+3)=depart(nusssn+3)+1
    endif
c
    return
    end
c
c*******************************************
c* Subroutine NEXTGUY                      *
c*******************************************
c
    subroutine nextguy(iat,inext)
    common/ucom1/ depart(10),rmean(10),p(10,10),servt(10),ecount(2)
    common/ucom2/ isubcap,nusssn,numcust,tclear,nstudy
c
    cum=0.
    u=unfrm(0.,1.,2)
c
    do 10 index=1,nusssn+2
      cum=cum+p(iat,index)
      if (u.le.cum) then
        inext=index
        goto 15
      else
        continue
      endif
10   continue
c
15   return
    end
c
c*******************************************
c* Subroutine OTPUT                        *
c*******************************************
c
    subroutine otput
    common/scom1/ atrib(100),dd(100),ddl(100),dtnow,ii,mfa,mstop,nclnr
    1,ncrdr,nprnt,nnrun,nnset,ntape,ss(100),ssl(100),tnext,tnow,xx(100)
    common/ucom1/ depart(10),rmean(10),p(10,10),servt(10),ecount(2)
    common/ucom2/ isubcap,nusssn,numcust,tclear,nstudy
```

```
      common/ucom3/ multino(7)
      common/ucom9/ level(10)
      real level
c
      write(15,*) ccavg(1), (level(i),i=1,nusssn+2)
c
      write(1,*)nnrun
      write(1,*)(ecount(i),i=1,2)
      write(1,*)(ccavg(i),i=1,nusssn+2)
      write(1,*)(ltavg(i),i=2,nusssn+2)
      write(1,*)(servt(i),i=1,nusssn+2)
      write(1,*)(depart(i),i=1,nusssn+3)
      isum=0
      do 10 i=1,7
        isum=isum+multino(i)
10    continue
      write(1,*)(multino(i),i=1,7),isum
c
      return
      end
```

```
5 5 25 2000. 1
0. 1. 0. 0. 0. 0. 0.
0. 0. 1. 0. 0. 0. 0.
.2 0. 0. .36 .36 .04 .04
0. 0. 1. 0. 0. 0. 0.
0. 0. 1. 0. 0. 0. 0.
0. 0. 1. 0. 0. 0. 0.
0. 0. 1. 0. 0. 0. 0.
GEN,bauer,model no 15,6/5/86,20,n,n,y,n,n;
LIMITS,7,5,200;
STAT,1,reponse time;
STAT,2,wait stat 2;
STAT,3,wait stat 3;
STAT,4,wait stat 4;
STAT,5,wait stat 5;
STAT,6,wait stat 6;
STAT,7,wait stat 7;
TIMST,XX(1),terminals;
TIMST,XX(2),cpu;
TIMST,XX(3),disk1;
TIMST,XX(4),disk2;
TIMST,XX(5),disk3;
TIMST,XX(6),disk4;
TIMST,XX(7),disk5;
INITIALIZE,0.,5000.;
MONTR,CLEAR,2000;
SEEDS,34444866917(1);
FIN;
```

Appendix 2. SAS Code for Obtaining β values

```
options linesize=78;
data;
infile fact;
input y x1 x2 x3 x4 x5 x6 x7;
x12=x1*x2;
x13=x1*x3;
x14=x1*x4;
x15=x1*x5;
x16=x1*x6;
x17=x1*x7;
x23=x2*x3;
x24=x2*x4;
x25=x2*x5;
x26=x2*x6;
x27=x2*x7;
x34=x3*x4;
x35=x3*x5;
x36=x3*x6;
x37=x3*x7;
x45=x4*x5;
x46=x4*x6;
x47=x4*x7;
x56=x5*x6;
x57=x5*x7;
x67=x6*x7;
proc stepwise;
model y = x1 x2 x3 x4 x5 x6 x7 x12 x13 x14 x15 x16 x17
          x23 x24 x25 x26 x27 x34 x35 x36 x37 x45 x46 x47
          x56 x57 x67;
```

Appendix 3. POST.FOR

```
C------------------------------------------------------------C
C This program takes a data file generated by M15.F and C
C computes candidate control variables for input into    C
C SELECT.II.F. Specifically, this program uses summed    C
C service times, probabilities of transitioning from     C
C the CPU to a particular node, the steady state         C
C probabilities of being in a particular node, and the   C
C actual number of customers transitioning from the CPU  C
C to a particular node, and calculates standardized      C
C work and routing variables.                            C
C------------------------------------------------------------C
   program mkdata
   parameter (numreps=20)
   integer dp
   real rm(7),pi(7),pi2(7)
   real r(13),w(7),e(8)
   integer mult(8)
   real wrkv(7),rmult(7)
   real ym(13),ymb(13)
   real wk(7),wk2(7),wkb(7),wkv(7)
   real rt(7),rt2(7),rtb(7),rtv(7)

c---------
c DATA FOR DESIGN POINT 1
c The following data are attributes of the model.
c---------

   data dp /1/
C rm are the mean service times by station
   data rm / 100.,0.,1.,2.224,2.224,20.,20./
C pi are the steady state transition probabilities
   data pi /.09,.09,.45,.16,.16,.02,.02/
C pi2 are the actual branching probabilities from CPU
   data pi2 / 2.,0.,0.,36.,36.,04.,04/

c---------
c dp1.dat is a data file generated by M15.F,
c the simulation of the model.
c---------

   open(unit=20,file='dp1.dat',status='old')
   do 10 i=1,numreps
C RUN #
    read(20,*)r1
C Total event count, Event count from TCLEAR
    read(20,*)r2,r3
```

```
C RESPONSES
    read(20,*)(r(i),i=1,7)
    read(20,*)(r(i),i=8,13)
C SUMS of Service Times by Station
    read(20,*)(w(i),i=1,7)
C TOTAL DEPARTURES by Station
    read(20,*)(e(i),i=1,8)
C Total Depatures from CPU to Station j
    read(20,*)(mult(i),i=1,8)

    do 20 j=1,7

c---------
c wrkv(j) is the standardized work variable of node j
c---------

       if(rm(j).ne.0.)then
     wrkv(j)=(w(j)-e(j)*rm(j))*(sqrt(e(j))/(pi(j)*e(8)*rm(j)))
     endif

c---------
c rmult(j) is the routing variable of node j
c---------

     if(j.eq.2.or.j.eq.3) then
      rmult(j)=0.
     else
      rmult(j)=(mult(j)-mult(8)*pi2(j))
   &  /sqrt(pi2(j)*(1.-pi2(j))*mult(8))
     endif

20   continue

c---------
c  ym(j) is the sum of response j
c---------

    do 30 j=1,13
     ym(j)=ym(j)+r(j)
30   continue

c---------
c wk(j) is the sum of work variable j
c wk2(j) is the sum of the squares of work variable j
c rt(j) is the sum of routing variable j
c rt2(j) is the sum of the squares of routing variable j
c---------

    do 40 j=1,7
```

```
      wk(j)=wk(j)+wrkv(j)
      wk2(j)=wk2(j)+wrkv(j)**2
      rt(j)=rt(j)+rmult(j)
      rt2(j)=rt2(j)+rmult(j)**2
40    continue

      open(unit=30,file='reg1.dat',status='new')
      write(30,*)r(1),r(9),r(10),r(11)
     &,r(12),r(13)
      write(30,*)wrkv(1),wrkv(3),wrkv(4)
     &,wrkv(5),wrkv(6),wrkv(7)
      write(30,*)rmult(1),rmult(4),rmult(5)
     &,rmult(6),rmult(7)

10    continue

c---------
c ymb(j) is the mean of response j
c---------

      do 50 j=1,13
       ymb(j)=ym(j)/(float(numreps))
50    continue

c---------
c wkb(j) is the mean of work variable j
c rtb(j) is the mean of routing variable j
c wkv(j) is the variance of work variable j
c rtv(j) is the variance of routing variable j
c---------

      do 60 j=1,7
       wkb(j)=wk(j)/(float(numreps))
       rtb(j)=rt(j)/(float(numreps))
60    continue

      do 70 j=1,7
        wkv(j)=(wk2(j)/(float(numreps-1))) -
     &((float(numreps/(numreps-1)))*wkb(j)**2)
       rtv(j)=(rt2(j)/(float(numreps-1))) -
     &((float(numreps/(numreps-1)))*rtb(j)**2)
70    continue

      open(unit=10,file='post1.out',status='new')
      write(10,555)
555   format(1x,'WELCOME TO TAPE10 : SUMMARY OF POST PROCESSING'/)
      write(10,560)dp
560   format(1x,'THIS IS DESIGN POINT ',i2,' POST PROCESSING'/)
      write(10,565)numreps
```

```
565  format(1x,'DP1 has ',i5,' replications'/)
     write(10,570)
570  format(1x,'Below are the sample means of the responses'/)
     write(10,*)ymb
     write(10,575)
575  format(1x,'Below are the means of the work variables'/)
     write(10,*)wkb
     write(10,580)
580  format(1x,'Below are the variances of the work variables'/)
     write(10,*)wkv
     write(10,585)
585  format(1x,'Below are the means of the routing variables'/)
     write(10,*)rtb
     write(10,590)
590  format(1x,'Below are the variances of the routing variables'/)
     write(10,*)rtv
     stop
     end
```

Appendix 4. SAS Code to Obtain Variance-Controlled Outputs

```
options linesize=78;
data;
infile reg1;
  input t u3 u4 u5 u6 u7 w1 w3 w4 w5 w6 w7 r1 r4 r5 r6 r7;
proc stepwise;
  model t = w1 w3 w4 w5 w6 w7 r1 r4 r5 r6;
proc stepwise;
  model u3 = w1 w3 w4 w5 w6 w7 r1 r4 r5 r6;
proc stepwise;
  model u4 = w1 w3 w4 w5 w6 w7 r1 r4 r5 r6;
proc stepwise;
  model u5 = w1 w3 w4 w5 w6 w7 r1 r4 r5 r6;
proc stepwise;
  model u6 = w1 w3 w4 w5 w6 w7 r1 r4 r5 r6;
proc stepwise;
  model u7 = w1 w3 w4 w5 w6 w7 r1 r4 r5 r6;
```

Appendix 5. SAS Code to Obtain Variance-Controlled Principal Components

```
options linesize=78;
data;
infile thesisb;
  input t u3 u4 u5 u6 u7 w1 w3 w4 w5 w6 w7 r1 r4 r5 r6 r7
         pc1 pc2 pc3 pc4 pc5 pc6;
proc stepwise;
  model pc1 = w1 w3 w4 w5 w6 w7 r1 r4 r5 r6;
proc stepwise;
  model pc2 = w1 w3 w4 w5 w6 w7 r1 r4 r5 r6;
proc stepwise;
  model pc3 = w1 w3 w4 w5 w6 w7 r1 r4 r5 r6;
proc stepwise;
  model pc4 = w1 w3 w4 w5 w6 w7 r1 r4 r5 r6;
```

Appendix 6. SAS Code for Factor Analysis of Original Outputs

```
options linesize=78;
data non;
infile thesisb;
input t u3 u4 u5 u6 u7 w1 w3 w4 w5 w6 w7 r1 r4 r5 r6 r7
       pc1 pc2 pc3 pc4 pc5 pc6;
proc factor data=non rotate=varimax score outstat=save;
  var t u3 u4 u5 u6 u7;
proc score data=non score=save out=scores;
proc plot;
  plot factor2*factor1;
proc plot;
  plot factor3*factor1;
proc plot;
  plot factor4*factor1;
proc print;
  var t u3 u4 u5 u6 u7 w1 w3 w4 w5 w6 w7 r1 r4 r5 r6 r7
       factor1 factor2 factor3 factor4;
```

Appendix 7. SAS Code for FActor Analysis of Variance-Controlled Outputs

```
options linesize=78;
data con;
infile step;
  input tb u3b u4b u5b u6b u7b;
proc factor data=con rotate=varimax score outstat=save;
  var tb u3b u4b u5b u6b u7b;
proc score data=con score=save out=scores;
proc plot;
  plot factor2*factor1;
proc plot;
  plot factor3*factor1;
proc plot;
  plot factor4*factor1;
proc print;
```

BIBLIOGRAPHY

Bauer, Kenneth W., Jr. Class Handout distributed in OPER 685, Applied Multivariate Analysis. School of Engineering, Air Force Institute of Technology (AU), Wright-Patterson AFB OH, January 1990.

-----. Class handout distributed in OPER 766, Advanced Topics in Simulation. School of Engineering, Air Force Institute of Technology (AU), Wright-Patterson AFB OH, August 1989.

-----. Control Variate Selection for Multiresponse Simulation. PhD Dissertation. School of Industrial Engineering, Purdue University, West Lafayette IN, 1987a.

Bauer, Kenneth W., Jr., Sekhar Venkatraman, and James R. Wilson. "Using Path Control Variates in Activity Network Simulation," Department of the Air Force AFIT/ENS Working Paper Series 88-01, January 1988.

-----. "Estimation Procedures Based on Control Variates With Known Covariance Matrix," 1987 Winter Simulation Conference Proceedings. 334-341. New York: IEEE Press,1987b.

Benski, H. Claudio. "Use of a Normality Test to Identify Significant Effects in Factorial Designs," Journal of Quality Technology, 21: 174-178 (July 1989).

Box, George E. P. and Norman R. Draper. Empirical Model-Building and Response Surface. New York: John Wiley & Sons, 1987.

Chatfield, Christopher and Alexander J. Collins. Introduction to Multivariate Analysis. New York: Chapman and Hall, 1980.

Dillon, William R. and Matthew Goldstein. Multivariate Analysis. New York: John Wiley & Sons, 1984.

Kaiser, Henry F. "The Varimax Criterion for Analytic Rotation in Factor Analysis," Psychometrika 23: 187-200 (September 1958).

Kleijnen, Jack P. C. Statistical Techniques in Simulation. New York: Marcel Dekker, Inc., 1974.

Lavenberg, S. S., T. L. Moeller, and P. D. Welch. Statistical Results on Control Variables with Application to Queueing Network Simulation," Operations Research 30:182-202 (1982).

Neter, John et al. Applied Linear Statistical Models. Illinois: Richard D. Irwin, Inc., 1985.

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The purpose of this research was to examine the effects of applying the technique of control variates on the principal components of a given model.

The investigation was done by comparing three sets of data as follows:

1) The set of principal components of the outputs of the model on which no variance reduction has been applied,

2) The set of principal components of variance-controlled outputs--control variates was performed prior to principal components analysis being done.

3) The set of variance-controlled principal components--control variates was performed on the principal components of the outputs.

The comparison of the effects was carried out by examining the percentage of variance explained by the principal components and by reviewing the scatter plots of the first two principal components.